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Compare the estimates of the reliability function and series system under Pareto distribution

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Abstract:

The two-parameter Pareto distribution was used to find the Reliability series system, With the Reliability function found for him, by comparing the estimation of distribution parameters and reliability functions, using the Statistical Scale (MAPE) and depending on the simulation method for MLE (maximum likelihood estimation) and non-parametric estimation (Empirical) methods, and it was found that the MSE statistic values for the non-parametric (Empirical) method were lower than the MLE method, and this indicates the advantage of the Empirical method.

Kew word: *Reliability function, Pareto distribution, Reliability series system, Simulation, matlap program.*

I. Introduction

The system is defined as a group of components, or subsystems, or units, in the form of a specific design in order to achieve the required mission, reliability of the system comes from the types of components, quantities, characteristics, and the way they are arranged inside the system. [1] [4]

Pareto distribution with two parameters (B: Scale and α: Shape parameters) in reliability applications, because it is one of the distributions of the failure function of models of stress and strength in mechanical engineering, it is attributed to the Italian economic researcher, (Vilfredo Pareto 1848-1923). [6]

II. Research objective

The research aims to compare the estimation parameters of the two-parameter Pareto distribution, and Reliability Function, as well as the Reliability series system (R.S.S), of two methods of estimating are: MLE, and the Empirical estimation, using the statistical measure (MSE).

III. Theoretical side

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3-1 Pareto distribution

A lot of research and studies have shown a complete explanation of the two-parameter Pareto distribution, and the methods for estimating the distribution parameters have been compared, and the distribution function can be illustrated as follows: [6] [3].

f(t)=α β^α t^(α-1) β≤t<∞ ,0<β<min(t_i),α>0 …(1)

3-2 Maximum Likelihood Estimation Method

If we have a random variable$(t_i\ i = 1,2, \ldots , n)$, and distributed as the Pareto distribution with two parameters (α, β), then to find the Maximum Likelihood Estimation Method by making the function at its maximum end and as follows [1] [2].

$$L(t_i, \alpha, \beta) = \alpha^n \beta^{n\alpha} \prod_{i=1}^{n} t_i^{-\alpha-1} \quad \ldots (2)$$

Taking the logarithm of equation (2), we get:

$$\ln L = n \ln(\alpha) + n\alpha \ln(\beta) - (\alpha + 1) \sum_{i=1}^{n} \ln(t_i) \quad \ldots (3)$$

By partial derivation of equation (3), and for the two parameters(α, β), equal to zero, it turns out that the estimator (β) cannot be obtained directly, and through the order of values (t_i), ascending we get:

$$\hat{\beta} = t_1 = \min(t_i) \quad i = 1,2, \ldots , n \quad \ldots (4)$$

The estimator (α) is as follows:

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{t_i}{\beta}\right)} \quad \ldots (5)$$

To obtain the reliability function, it is as follows:

$$R(t) = \int_t^{\infty} f(y)dy = \int_t^{\infty} \alpha \beta^{\alpha} y^{\alpha-1} dy \quad \ldots (6)$$

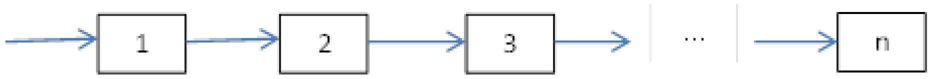
So

$$R(t) = \left(\frac{\beta}{t}\right)^{\alpha} \quad \ldots (7)$$

The estimators of the Maximum Likelihood Estimation can be used to obtain the reliability function as follows:

$$\hat{R}(t) = \left(\frac{t_1}{t}\right)^{\frac{n}{\sum_{i=1}^{n} \ln\left(\frac{t_i}{t_1}\right)}} \quad \ldots (8)$$

In order to find a series system, respectively, each unit has a Pareto distribution according to the following form:[6]



Where the system is in the state of (i.i.d.), then:

$$R_S(t) = [R(t)]^n = [(\frac{\beta}{t})^{\alpha}]^n \qquad ... (9)$$

And

$$F_S(t) = 1 - [R(t)]^n = 1 - (\frac{\beta}{t})^{n\alpha} \qquad ... (10)$$

To find the distribution of the system it can be obtained as follows:

$$f_S(t) = 0 - \mathrm{n}[R(t)]^{n-1} \frac{d}{dt} R(t) \qquad ... (11)$$

$$f_S(t) = -\mathrm{n} (\frac{\beta}{t})^{\alpha(n-1)} \frac{d}{dt} (\frac{\beta}{t})^{\alpha} \qquad ... (12)$$

$$f_S(t) = -\mathrm{n} (\frac{\beta}{t})^{\alpha(n-1)} \frac{-\alpha t^{\alpha-1} \beta^{\alpha}}{t^{2\alpha}} \qquad ... (13)$$

$$f_S(t)_{Ml} = \frac{n\, \alpha_{ML} \beta_{ML}{}^{n\, \alpha_{ML}}}{t^{\alpha_{ML}(n-1)}} \qquad ... (14)$$

To find the system failure ratio function ($HR_S(t)$), it can be found as follows:

$$h_S(t) = \frac{f_S(t)}{R_S(t)} \qquad ... (15)$$

$$h_S(t) = \frac{n\, \alpha_{ML}}{t^{-\alpha_{ML}}} = HR_S(t) \qquad ... (16)$$

3-3 The Empirical or non-parametric method depends on the maximum likelihood

This method is known (Empirical or Non-Parametric or free Distribution), through the reliability function, then: [5].

$$\hat{R}(t) = \frac{n+1-i}{n+1} \qquad ... (17)$$

$$\hat{R}(t_1) = \frac{n}{n+1} \qquad ... (18)$$

So we get:

$$\frac{n}{n+1} = (\frac{\beta}{t_1})^{\hat{\alpha}} \qquad ... (19)$$

Taking the logarithm of both sides of equation (19), we get:

$$\ln (\frac{n}{n+1}) = \hat{\alpha}\ln(\beta) - \hat{\alpha}\ln(t_1) \qquad ... (20)$$

From equation (5) we obtain:

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$$\hat{\alpha}[\sum \ln(t_i) - n\ln(\beta)] = n \qquad \ldots(21)$$

$$\therefore \ln(\beta) = \frac{\hat{\alpha}\sum \ln(t_i) - n}{\hat{\alpha}\, n} \qquad \ldots(22)$$

And by substitute equation 22 with equation 20 we obtained the follow:

$$\ln\left(\frac{n}{n+1}\right) = \hat{\alpha}\left(\frac{\hat{\alpha}\sum \ln(t_i) - n}{\hat{\alpha} n}\right) - \hat{\alpha}\ln(t_1) \qquad \ldots(23)$$

$$\hat{\alpha}_{EM} = \frac{n\left[1+\ln\left(\frac{n}{n+1}\right)\right]}{\sum \ln(t_i) - n\ln(t_1)} \qquad \ldots(24)$$

And by substitute equation 24 with equation 22 we obtained the follow:

$$\therefore \ln(\beta) = \frac{n\left[1+\ln\left(\frac{n}{n+1}\right)\right]\sum \ln(t_i) - n[\sum \ln(t_i) - n\ln(t_1)]}{n\left\{n\left[1+\ln\left(\frac{n}{n+1}\right)\right]\right\}} \qquad \ldots(25)$$

$$\beta_{EM} = \mathrm{EXP}\left[\frac{\sum \ln(t_i)\ln\left(\frac{n}{n+1}\right) - n\ln(t_1)}{n\left[1+\ln\left(\frac{n}{n+1}\right)\right]}\right] \qquad \ldots(26)$$

The reliability function of the non-parametric method is as follows:

$$\hat{R}_{EM}(t) = \left(\frac{\beta_{EM}}{t}\right)^{\hat{\alpha}_{EM}} \qquad \ldots(27)$$

And that the distribution of the series system, respectively, and each unit has a Pareto distribution according to the non-parametric method estimate is as follows:

$$f_S(t)_{EM} = \frac{n\,\alpha_{EM}\,\beta_{EM}{}^{n\,\alpha_{EM}}}{t^{\alpha_{EM}(n-1)}} \qquad \ldots(28)$$

To find the system failure ratio function ($HR_S(t)$), it can be obtained as in equation (15) as follows:[5]

$$h_S(t)_{EM} = \frac{n\,\alpha_{ML}}{t^{-\alpha_{ML}}} = HR_S(t) \qquad \ldots(29)$$

IV. Simulation

Five simulation experiments are designed to repeat 1000 times, to sample sizes (n = 10, 20, 30, 40, 50) and for initial parameter values $(\alpha_0 = 0.5, 1, 1.2, 1.5, 2.5)$, $(\beta_0 = 0.2, 0.4, 0.5, 0.8, 1)$, And based on initial values of failure time Pareto parameters with two parameters, reliability function as well as reliability function were estimated as a series system, respectively ($t_0 = 1.25, 3.25, 5, 7.8, 9$) The MSE was calculated for all estimates.

Table No. (1) Shows the estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, for two estimated methods and calculating the MSE statistic for each estimate and for the first experience

		Estimation								MSE							
.5	.2	MLE				Emparcal E				MLE				Emparcal E			
		α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2	α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2
	.25	0 .626661	1 .242421	0 .992872	7 .172504	0 .566934	0 .667522	0 .707602	0 .054621	0 .442019	1 .617503	1 .005346	8 159.711	0 .361776	0 .463595	0 .51078	0 .005135
	.25	0 .629216	1 .235018	0 .555195	0 .030295	0 .569246	0 .674436	0 .428003	0 .00125	0 .454843	1 .62084	0 .325971	0 .13321	0 .372273	0 .473021	0 .196952	7 .15E-06
0		0 .614562	1 .258113	0 .444786	0 .003272	0 .555988	0 .661925	0 .347221	0 .000305	0 .432801	1 .690017	0 .214293	0 .000255	0 .354232	0 .457907	0 .133277	7 .28E-07
	.8	0 .626981	1 .263115	0 .34141	0 .000862	0 .567223	0 .663715	0 .272744	7 .15E-05	0 .447194	1 .728014	0 .1315	0 .000122	0 .366012	0 .460398	0 .085872	1 .12E-07
		0 .613287	1 .27294	0 .324808	0 .000401	0 .554835	0 .656163	0 .259783	4 .31E-05	0 .429666	1 .750941	0 .120204	4 .26E-06	0 .351666	0 .450963	0 .078273	2 .33E-08
		0	1	0	1	0	0	0	0	0	1	0	9	0	0	0	0

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0	.25	.553637	.109917	.935065	.114875	.526625	.817782	.801501	.0202	.323687	.246462	.877497	0.41721	.292872	.67558	.645257	.000698
	.25	0 .553255	1 .102571	0 .554227	0 .000102	0 .526261	0 .822757	0 .491395	1 .03E-05	0 .321873	1 .227817	0 .313103	4 .36E-07	0 .291231	0 .683129	0 .24715	1 .7E-09
		0 .55234	1 .107869	0 .442814	4 .06E-06	0 .525391	0 .818932	0 .395651	6 .6E-07	0 .322718	1 .242343	0 .203208	6 .39E-10	0 .291995	0 .677179	0 .163321	1 .25E-11
	.8	0 .552934	1 .110535	0 .350976	2 .39E-07	0 .525956	0 .816883	0 .316616	2 .49E-08	0 .324251	1 .247454	0 .130484	2 .94E-12	0 .293382	0 .674168	0 .106536	1 .76E-14
		0 .554924	1 .111122	0 .323935	5 .19E-08	0 .527849	0 .817106	0 .293183	7 .62E-09	0 .325522	1 .249461	0 .111552	1 .56E-13	0 .294533	0 .674472	0 .091876	2 .13E-15
	.25	0 .533676	1 .072419	0 .921228	0 .564314	0 .516177	0 .873271	0 .831753	0 .00725	0 .295701	1 .155944	0 .850063	1 17.9246	0 .276627	0 .766161	0 .693361	9 .74E-05
	.25	0 .538682	1 .070091	0 .552871	7 .38E-07	0 .521019	0 .875329	0 .508896	6 .86E-08	0 .300645	1 .150078	0 .309618	2 .35E-11	0 .281252	0 .769278	0 .262882	2 E-13
0		0 .537919	1 .073798	0 .442751	1 .95E-08	0 .52028	0 .872175	0 .409359	1 .38E-09	0 .300939	1 .158507	0 .201063	1 .68E-13	0 .281527	0 .764096	0 .17241	1 .75E-16
	.8	0 .539504	1 .07628	0 .350166	1 .59E-10	0 .521813	0 .871076	0 .325644	8 .65E-12	0 .301975	1 .16487	0 .127123	1 .49E-17	0 .282496	0 .762592	0 .110237	1 .51E-20
		0	1	0	3	0	0	0	3	0	1	0	6	0	0	0	3

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		.532116	.073692	.329647	.59E-11	.514668	.872486	.308162	.42E-12	.293523	.159253	.113343	.62E-19	.274589	.765081	.099086	.08E-21
	.25	0 .527191	1 .051058	0 .912367	0 .073366	0 .514173	0 .905592	0 .84792	0 .002472	0 .285478	1 .107431	0 .83317	0 .150116	0 .271553	0 .822038	0 .719761	1 .27E-05
	.25	0 .526344	1 .055429	0 .555673	5 .39E-09	0 .513347	0 .901826	0 .520786	4 .31E-10	0 .28467	1 .116995	0 .311769	1 .21E-15	0 .270785	0 .815395	0 .274201	5 .59E-18
0		0 .52924	1 .055595	0 .442962	1 .78E-11	0 .516172	0 .902288	0 .417498	1 .24E-12	0 .288004	1 .118025	0 .199789	5 .16E-20	0 .273957	0 .816476	0 .177764	9 .89E-23
	.8	0 .525431	1 .051606	0 .353822	9 .96E-15	0 .512457	0 .905293	0 .336742	3 .44E-15	0 .283401	1 .10908	0 .128652	7 .87E-27	0 .269578	0 .821698	0 .116684	1 .68E-27
		0 .527235	1 .05312	0 .327903	4 .27E-15	0 .514216	0 .904006	0 .312141	3 .39E-16	0 .285422	1 .112125	0 .110885	9 .57E-27	0 .2715	0 .819331	0 .100644	2 .1E-29
	.25	0 .520877	1 .04225	0 .909563	0 .037342	0 .510562	0 .922423	0 .856568	0 .00085	0 .276714	1 .088336	0 .82787	0 .052638	0 .265863	0 .852352	0 .734323	1 .62E-06
0	.25	0 .524374	1 .042468	0 .552825	3 .39E-11	0 .51399	0 .922294	0 .525664	2 .71E-12	0 .280806	1 .088602	0 .307948	1 .31E-19	0 .269794	0 .852026	0 .278669	5 .96E-22
		0 .526298	1 .043053	0 .441405	1 .76E-14	0 .515876	0 .922078	0 .421423	1 .35E-15	0 .283114	1 .090112	0 .197717	1 .06E-25	0 .272012	0 .851787	0 .180279	2 .64E-28
		0	1	0	7	0	0	0	1	0	1	0	2	0	0	0	4

	.8	.518819	.03933	.355331	.01E-18	.508545	.924603	.342093	.36E-18	.274843	.081827	.128974	.09E-32	.264066	.856106	.119678	.71E-34
		0.521905	1.042653	0.328671	6.04E-19	0.51157	0.922232	0.315886	2.53E-19	0.277893	1.08938	0.110683	1.06E-34	0.266996	0.852074	0.102306	3.71E-35

Table No. (2) Shows the estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, for two estimated methods and calculating the value of the MSE statistic for each estimate and for the second experiment.

		Estimation								MSE							
	.4	MLE				Empirical E				MLE				Empirical E			
		α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2	α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2
	.25	1.270898	1.115895	0.871761	13.351	1.149769	0.810629	0.621667	0.019581	1.831844	1.263816	0.785657	77264.3	1.499298	0.66647	0.399221	0.000829
0	.25	1.251162	1.110848	0.290202	0.000231	1.131913	0.811706	0.237634	2.78E-05	1.797508	1.249057	0.098093	4.17E-06	1.471195	0.667148	0.066944	1.69E-08
		1.243473	1.114927	0.185215	1.15E-05	1.124957	0.808318	0.157916	2.38E-06	1.750296	1.258351	0.04406	4.18E-09	1.432553	0.662054	0.032342	1.46E-10

	.8	1 .235306	1 .114796	0 .117646	6 .75E-07	1 .117569	0 .807968	0 .10463	1 .59E-07	1 .710952	1 .257043	0 .019438	6 .79E-11	1 .400352	0 .661065	0 .015341	1 .61E-12
		1 .23151	1 .10627	0 .103633	4 .08E-07	1 .114135	0 .813788	0 .094947	2 .07E-07	1 .729203	1 .23678	0 .015706	1 .36E-11	1 .41529	0 .66992	0 .013289	2 .72E-12
	.25	1 .126136	1 .05476	0 .826897	0 .256759	1 .071192	0 .901341	0 .709186	0 .00268	1 .363354	1 .115955	0 .688432	2 8.97233	1 .233563	0 .815025	0 .507258	1 .87E-05
	.25	1 .110466	1 .051124	0 .29711	3 .29E-08	1 .056286	0 .904235	0 .270804	4 .1E-09	1 .300633	1 .107961	0 .094811	2 .19E-13	1 .176813	0 .819927	0 .079031	1 .5E-15
0		1 .106422	1 .051073	0 .19252	2 .24E-10	1 .05244	0 .903755	0 .179581	3 .05E-11	1 .296869	1 .107545	0 .042158	8 .46E-18	1 .173408	0 .818944	0 .036853	9 .9E-20
	.8	1 .102683	1 .050498	0 .123455	5 .41E-13	1 .048883	0 .904377	0 .117749	2 .44E-13	1 .286487	1 .106382	0 .018286	1 .6E-22	1 .164013	0 .820162	0 .01658	4 .02E-23
		1 .10798	1 .054839	0 .107367	1 .03E-12	1 .053922	0 .90072	0 .102288	3 .53E-13	1 .302841	1 .116056	0 .014348	9 .48E-22	1 .178811	0 .813856	0 .01303	1 .01E-22
	.25	1 .075036	1 .032039	0 .814491	0 .007703	1 .039785	0 .936837	0 .742754	0 .000378	1 .197346	1 .066193	0 .66522	0 .000962	1 .120112	0 .878595	0 .553622	4 .88E-07
0	.25	1 .066999	1 .035244	0 .30264	4 .21E-12	1 .032013	0 .933724	0 .283878	3 .05E-13	1 .179539	1 .073069	0 .095952	5 .26E-21	1 .103453	0 .872936	0 .084684	7 .4E-24

		1.074529	1.034435	0.193695	1.32E-15	1.039296	0.934628	0.184572	4E-16	1.199404	1.071347	0.041074	3.75E-28	1.122037	0.87464	0.037286	4.01E-29
	.8	1.070795	1.03259	0.123595	3.02E-19	1.035683	0.936225	0.120043	9.77E-20	1.186477	1.06733	0.017344	7.38E-35	1.109944	0.877452	0.016373	6.91E-36
		1.070517	1.033987	0.107978	1.41E-20	1.035415	0.934863	0.105055	7.3E-21	1.188395	1.070304	0.013611	3.51E-38	1.111738	0.874964	0.012871	1.91E-38
	.25	1.053976	1.025332	0.812137	0.001175	1.02795	0.951005	0.756013	4.79E-05	1.138944	1.051991	0.660722	4.8E-05	1.083391	0.905001	0.572939	1.01E-08
	.25	1.044197	1.023704	0.304579	5.15E-17	1.018414	0.952306	0.291835	1.69E-17	1.117594	1.048551	0.095857	1.52E-31	1.063083	0.907404	0.088081	2.42E-32
0		1.047987	1.026639	0.19688	1.51E-21	1.02211	0.949608	0.189624	3.77E-22	1.127161	1.05469	0.04119	5.68E-40	1.072183	0.902364	0.038262	5.65E-41
	.8	1.047383	1.025467	0.126049	1.1E-25	1.02152	0.950791	0.122996	3.58E-26	1.12493	1.052336	0.0175	8.25E-48	1.070061	0.904655	0.016638	9.94E-49
		1.056716	1.025426	0.10749	4.66E-28	1.030623	0.951036	0.105298	2.67E-28	1.146759	1.052276	0.012912	1.26E-52	1.090826	0.905135	0.012385	2.56E-53
0	.25	1.038533	1.021658	0.811304	0.000249	1.017967	0.959198	0.764569	6.5E-06	1.100677	1.044279	0.659151	3.92E-06	1.057516	0.920506	0.585556	3.3E-10

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	.25	1 .050103	1 .019124	0 .300857	1 .56E-20	1 .029308	0 .961727	0 .290616	1 .74E-21	1 .128729	1 .038976	0 .093346	1 .05E-37	1 .084468	0 .925247	0 .087201	1 .23E-39
		1 .039766	1 .020639	0 .197358	1 .05E-26	1 .019176	0 .960177	0 .191742	7 .08E-27	1 .105809	1 .042188	0 .041092	5 .59E-50	1 .062446	0 .92237	0 .038786	3 .2E-50
	.8	1 .043236	1 .020321	0 .125364	4 .57E-31	1 .022577	0 .960533	0 .122971	1 .26E-31	1 .111323	1 .041498	0 .01709	1 .9E-58	1 .067744	0 .92302	0 .016441	1 .43E-59
		1 .033071	1 .019379	0 .110875	1 .74E-34	1 .012613	0 .961198	0 .109288	1 .4E-34	1 .089747	1 .039498	0 .013486	2 .3E-65	1 .047015	0 .924231	0 .013107	1 .88E-65

Table No. (3) Shows the estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, for two estimated methods and calculating the value of the MSE statistic for each estimate and for the third experience

		Estimation								MSE							
.2	.5	MLE				Empirical E				MLE				Empirical E			
		α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2	α_1	β_1	R t1	R ts1	α_2	β_2	R t2	R ts2
		1	1	0	0	1	0	0	0	2	1	0	9	2	0	0	0

0	.25	.522382	.092913	.821908	.536848	.377284	.838358	.591516	.013281	.632636	.20328	.690598	.881931	.154717	.708856	.362716	.000423
	.25	1 .480394	1 .086528	0 .23008	3 .34E-05	1 .339297	0 .841209	0 .195746	1 .04E-05	2 .527926	1 .188739	0 .064698	2 .5E-08	2 .069015	0 .713249	0 .047782	3 .9E-09
		1 .484023	1 .090054	0 .134372	1 .22E-06	1 .34258	0 .839305	0 .119507	4 .97E-07	2 .529567	1 .197376	0 .024461	1 .09E-10	2 .070359	0 .710392	0 .019516	2 .04E-11
	.8	1 .52099	1 .089437	0 .075013	6 .74E-08	1 .376024	0 .841899	0 .069888	2 .53E-08	2 .60575	1 .196331	0 .009189	3 .26E-13	2 .132711	0 .715016	0 .007873	4 .76E-14
		1 .466726	1 .097113	0 .06785	1 .45E-08	1 .326932	0 .83453	0 .063135	6 .89E-09	2 .403848	1 .21595	0 .007322	1 .67E-14	1 .967462	0 .703394	0 .006289	5 .14E-15
	.25	1 .323456	1 .043367	0 .788474	0 .03577	1 .258885	0 .917937	0 .682337	0 .001361	1 .851199	1 .090648	0 .625763	0 .031381	1 .674965	0 .84435	0 .469531	5 .45E-06
	.25	1 .336007	1 .044447	0 .233022	1 .34E-09	1 .270823	0 .917244	0 .213898	2 .93E-10	1 .888388	1 .092915	0 .060027	1 .7E-16	1 .708614	0 .84306	0 .050785	1 .18E-17
0		1 .342053	1 .040711	0 .136204	2 .57E-12	1 .276574	0 .92061	0 .129394	5 .88E-13	1 .908461	1 .084674	0 .022154	1 .3E-21	1 .726776	0 .848915	0 .019974	3 .34E-23
	.8	1 .340931	1 .044699	0 .079924	4 .77E-15	1 .275507	0 .917312	0 .077327	2 .09E-15	1 .895851	1 .093627	0 .008288	6 .1E-27	1 .715366	0 .843246	0 .007751	1 .9E-27
		1	1	0	6	1	0	0	6	1	1	0	3	1	0	0	3

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		.346458	.043828	.067044	.06E-15	.280764	.917967	.065485	.39E-15	.911392	.091436	.006101	.31E-26	.729428	.84422	.005799	.9E-26
	.25	1 .292628	1 .027841	0 .777518	0 .002478	1 .250243	0 .945873	0 .707867	0 .000117	1 .731807	1 .057265	0 .606682	0 .000216	1 .620098	0 .895404	0 .503387	7 .02E-08
	.25	1 .286429	1 .029573	0 .236786	6 .52E-14	1 .244247	0 .944146	0 .223603	1 .48E-14	1 .715605	1 .060968	0 .06011	6 .41E-25	1 .604941	0 .892257	0 .05364	4 .41E-26
0		1 .287047	1 .028033	0 .14006	2 .89E-18	1 .244845	0 .945602	0 .135139	6 .96E-19	1 .718198	1 .057732	0 .022079	3 .18E-33	1 .607367	0 .894933	0 .020581	1 .92E-34
	.8	1 .286568	1 .027456	0 .082033	5 .56E-24	1 .244381	0 .946123	0 .080643	2 .02E-24	1 .712291	1 .056453	0 .008066	4 .44E-45	1 .601841	0 .895853	0 .007784	4 .79E-46
		1 .27886	1 .02944	0 .071766	4 .6E-24	1 .236926	0 .94399	0 .070526	4 .37E-24	1 .702796	1 .060636	0 .006417	4 .47E-45	1 .592958	0 .891926	0 .006176	6 .33E-45
	.25	1 .266439	1 .02103	0 .774642	0 .000203	1 .235167	0 .958906	0 .722178	9 .91E-06	1 .646712	1 .04291	0 .601492	1 .72E-06	1 .566393	0 .919885	0 .523074	4 .9E-10
0	.25	1 .259463	1 .020584	0 .239813	1 .17E-18	1 .228363	0 .959221	0 .230738	7 .12E-19	1 .636323	1 .042064	0 .06068	7 .42E-34	1 .556511	0 .920527	0 .056309	3 .18E-34
		1 .255558	1 .021735	0 .142865	2 .8E-24	1 .224555	0 .958117	0 .138864	2 .59E-24	1 .61667	1 .044421	0 .022251	6 .49E-45	1 .537816	0 .918425	0 .021044	6 .05E-45
		1	1	0	8	1	0	0	3	1	1	0	4	1	0	0	6

	.8	.265135	.021125	.08304	.28E-32	.233896	.958826	.081829	.91E-32	.64578	.043156	.007918	.87E-60	.565506	.919776	.007675	.34E-61
		1 .254496	1 .021861	0 .071718	2 .33E-33	1 .22352	0 .957929	0 .070868	1 .82E-33	1 .61788	1 .044664	0 .006028	1 .34E-63	1 .538967	0 .918056	0 .005878	2 .33E-63
	.25	1 .259784	1 .017372	0 .772057	2 .3E-05	1 .234837	0 .966562	0 .729053	9 .21E-07	1 .622868	1 .035342	0 .597254	2 .41E-08	1 .55923	0 .934522	0 .532746	7 .82E-12
	.25	1 .253023	1 .016756	0 .238613	1 .31E-23	1 .22821	0 .96711	0 .231136	1 .42E-24	1 .60585	1 .034124	0 .059432	4 .82E-44	1 .54288	0 .935605	0 .055799	5 .26E-46
0		1 .248271	1 .016898	0 .142553	9 .09E-30	1 .223552	0 .966924	0 .13946	5 .98E-30	1 .591075	1 .034405	0 .021883	8 .17E-56	1 .528684	0 .935243	0 .020931	3 .55E-56
	.8	1 .256353	1 .016615	0 .082414	3 .25E-39	1 .231474	0 .967273	0 .081601	1 .85E-39	1 .611343	1 .033791	0 .007609	9 .38E-75	1 .548157	0 .935875	0 .007463	3 .08E-75
		1 .252534	1 .017065	0 .069774	4 .09E-39	1 .227731	0 .966834	0 .069185	5 .04E-41	1 .599488	1 .034738	0 .005509	1 .65E-74	1 .536767	0 .935062	0 .005409	2 .15E-78

Table No. (4) Shows the estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, for two estimated methods and calculating the value of the MSE statistic for each estimate and for the fourth experiment

		Estimation	MSE

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.5	.8	MLE				Empirical E				MLE				Empirical E			
		α_1	β_1	Rt1	Rts1	α_2	β_2	Rt2	Rts2	α_1	β_1	Rt1	Rts1	α_2	β_2	Rt2	Rts2
	.25	1.871768	1.07175	0.758736	0.762351	1.69337	0.867841	0.558655	0.008495	4.156153	1.154458	0.593759	120.9116	3.40166	0.757755	0.325138	0.000196
	.25	1.886152	1.069675	0.15094	2.25E-06	1.706382	0.870649	0.132272	8.28E-07	3.99512	1.148944	0.029699	3.74E-10	3.26986	0.761868	0.023056	6.81E-11
0		1.889456	1.069404	0.083458	1.29E-07	1.709371	0.869893	0.077569	4.39E-08	4.152101	1.148368	0.011044	2.42E-12	3.398344	0.760853	0.009486	1.51E-13
	.8	1.900888	1.071013	0.044112	7.13E-09	1.719714	0.869423	0.043327	6.36E-09	4.166234	1.15276	0.003843	1.2E-14	3.409911	0.760485	0.003631	1.11E-14
		1.864675	1.070887	0.036786	2.31E-09	1.686953	0.868736	0.036854	1.6E-09	3.947633	1.152101	0.00268	3E-15	3.230995	0.759035	0.00261	1.2E-15
0	.25	1.663554	1.035457	0.734251	0.146613	1.582389	0.932888	0.634274	0.000453	2.924189	1.073634	0.543896	18.29673	2.645807	0.871534	0.40753	8.77E-07
	.25	1.669658	1.032629	0.161994	1.68E-11	1.588195	0.935504	0.152686	8.75E-12	2.951274	1.06746	0.0305	6E-20	2.670313	0.876189	0.027205	2.68E-20

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		1.67566	1.034983	0.084507	2.11E-15	1.593904	0.933516	0.081604	1.41E-15	2.974471	1.072545	0.009088	1.43E-27	2.691302	0.872659	0.008471	7.97E-28
	.8	1.679254	1.035519	0.044858	2.24E-18	1.597323	0.932826	0.044515	1.85E-18	2.996328	1.073544	0.002989	1.25E-33	2.711078	0.871323	0.002917	1.75E-33
		1.657704	1.036416	0.037576	6.83E-20	1.576824	0.931722	0.037598	9E-20	2.914002	1.075514	0.002117	7.51E-37	2.636589	0.869334	0.002097	2.43E-36
	.25	1.601675	1.022819	0.726948	0.00041	1.549156	0.955495	0.662221	2.77E-05	2.663528	1.046669	0.531062	3.98E-06	2.491718	0.913446	0.441441	6.24E-09
	.25	1.616239	1.02198	0.163774	4.33E-17	1.563243	0.956495	0.157203	2.33E-17	2.709583	1.044899	0.029831	3.07E-31	2.534803	0.915331	0.027443	2.21E-31
0		1.596376	1.022648	0.088419	3.66E-22	1.544031	0.955653	0.086526	2.64E-22	2.644559	1.046356	0.009305	4.8E-41	2.473973	0.913782	0.008885	3.51E-41
	.8	1.608219	1.023765	0.045273	6.92E-27	1.555486	0.954716	0.045124	1.78E-27	2.681988	1.048656	0.002674	4.74E-50	2.508988	0.912002	0.002646	3E-51
		1.60245	1.022729	0.037189	1.87E-26	1.549906	0.955672	0.03745	2.72E-26	2.662558	1.046539	0.001878	3.51E-49	2.490811	0.913829	0.001894	7.4E-49
0	.25	1.584131	1.016069	0.72145	1.94E-05	1.545015	0.967615	0.675056	1.38E-06	2.581562	1.032648	0.522299	8.8E-09	2.455645	0.936522	0.457765	1.78E-11

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	.25	1 .577076	1 .017209	0 .166611	1 .7E-21	1 .538134	0 .966483	0 .161285	4 .94E-23	2 .551004	1 .034988	0 .029769	2 .8E-39	2 .426578	0 .934354	0 .027916	1 .21E-42
		1 .57666	1 .017147	0 .087751	5 .78E-29	1 .537728	0 .966545	0 .086364	3 .97E-30	2 .553397	1 .034894	0 .008759	3 .33E-54	2 .428854	0 .934498	0 .008482	1 .56E-56
	.8	1 .579271	1 .017283	0 .045503	1 .71E-39	1 .540275	0 .966449	0 .045466	1 .94E-39	2 .561652	1 .035194	0 .002556	2 .01E-75	2 .436706	0 .934335	0 .002542	2 .87E-75
		1 .582795	1 .016046	0 .036328	2 .92E-41	1 .543711	0 .967683	0 .036622	6 .6E-41	2 .569657	1 .032612	0 .001653	6 .81E-79	2 .444321	0 .936664	0 .001674	4 .3E-78
	.25	1 .552894	1 .01332	0 .722595	1 .22E-06	1 .522143	0 .973508	0 .68469	6 .54E-08	2 .458533	1 .026993	0 .523421	4 .33E-11	2 .362126	0 .947889	0 .470135	6 .7E-14
	.25	1 .567406	1 .013488	0 .166363	3 .73E-29	1 .536367	0 .973476	0 .162247	2 .21E-29	2 .507308	1 .027363	0 .029404	4 .28E-55	2 .408989	0 .947851	0 .027984	1 .55E-55
0		1 .569135	1 .013402	0 .087289	3 .26E-39	1 .538062	0 .973533	0 .086233	3 .01E-39	2 .51776	1 .027166	0 .008554	5 .53E-75	2 .419031	0 .947945	0 .008342	5 .87E-75
	.8	1 .558138	1 .01324	0 .045923	5 .34E-46	1 .527283	0 .973619	0 .04598	9 .13E-46	2 .479841	1 .026841	0 .002502	2 .82E-88	2 .382599	0 .948111	0 .002505	8 .24E-88
		1 .578397	1 .014049	0 .035899	5 .67E-50	1 .547141	0 .972979	0 .036021	5 .82E-50	2 .546793	1 .02849	0 .001572	3 .21E-96	2 .446925	0 .946876	0 .001579	3 .38E-96

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Table No. (5) Shows the estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, for two estimated methods and calculating the value of the MSE statistic for each estimate and for the fifth experience

		Estimation								MSE							
.5		MLE				Empirical E				MLE				Empirical E			
		α_1	β_1	Rt1	Rts1	α_2	β_2	Rt2	Rts2	α_1	β_1	Rt1	Rts1	α_2	β_2	Rt2	ts2
	.25	3.067369	1.040484	0.584132	0.034937	2.775018	0.918913	0.447886	0.001692	10.70115	1.084442	0.358285	0.089018	8.758503	0.846299	0.214043	1.22E-05
	.25	3.143394	1.042446	0.0496	9.19E-09	2.843796	0.917804	0.047935	5.21E-09	11.24887	1.088645	0.004502	1.7E-14	9.206788	0.844387	0.004115	4.9E-15
0		3.125987	1.041305	0.017837	4.57E-11	2.828049	0.919046	0.018974	1.52E-11	11.02472	1.086064	0.000768	1.16E-18	9.023335	0.846488	0.000816	1.01E-19
	.8	3.156627	1.041417	0.00759	1.04E-11	2.855768	0.918811	0.008665	7.93E-12	11.42486	1.086383	0.000215	1.06E-19	9.350835	0.846157	0.000254	6.03E-20
		3.058644	1.040492	0.00576	4.62E-13	2.767124	0.919145	0.006784	8.32E-13	10.72476	1.084604	0.000113	2.08E-22	8.777828	0.846822	0.000143	6.8E-22
		2	1	0	0	2	0	0	1	7	1	0	3	7	0	0	3

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0	.25	.746938	.020007	.577539	.000239	.612915	.959731	.508789	.69E-05	.98179	.040807	.339751	.23E-06	.221925	.921493	.26491	.89E-09
	.25	2 .758254	1 .019995	0 .051372	1 .03E-17	2 .623679	0 .959893	0 .050918	8 .37E-19	8 .028487	1 .040792	0 .003657	8 .43E-32	7 .264177	0 .921826	0 .003545	8 .78E-35
		2 .7769	1 .021136	0 .018683	2 .62E-23	2 .641415	0 .958802	0 .019365	5 .91E-23	8 .15276	1 .04316	0 .000609	3 .21E-43	7 .376619	0 .919756	0 .000645	1 .56E-42
	.8	2 .789157	1 .019903	0 .006566	1 .07E-26	2 .653074	0 .960166	0 .007165	3 .11E-26	8 .18929	1 .040589	9 .8E-05	1 .14E-49	7 .409671	0 .92232	0 .000114	9 .67E-49
		2 .759413	1 .018831	0 .004986	3 .76E-31	2 .624781	0 .961104	0 .005517	1 .39E-30	8 .025291	1 .038431	5 .83E-05	9 .38E-59	7 .261284	0 .92414	6 .85E-05	1 .64E-57
	.25	2 .690596	1 .013195	0 .572141	1 .61E-06	2 .602372	0 .973467	0 .526486	1 .45E-07	7 .535462	1 .026744	0 .33158	5 .02E-11	7 .049391	0 .947824	0 .281445	4 .37E-13
	.25	2 .665489	1 .013341	0 .052114	2 .83E-26	2 .578088	0 .973247	0 .051923	2 .56E-27	7 .360426	1 .027044	0 .003454	7 .24E-49	6 .885646	0 .947399	0 .003413	4 .09E-51
0		2 .685181	1 .013382	0 .018695	9 .97E-35	2 .597134	0 .973252	0 .019215	1 .28E-34	7 .500368	1 .027133	0 .000549	4 .65E-66	7 .016561	0 .947415	0 .000571	1 .3E-65
	.8	2 .655407	1 .013608	0 .00691	2 .5E-44	2 .568337	0 .972909	0 .007333	3 .76E-44	7 .312063	1 .027586	9 .15E-05	3 .46E-85	6 .840402	0 .94674	0 .000101	1 .13E-84
		2	1	0	1	2	0	0	9	7	1	4	1	6	0	5	9

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		.678872	.013089	.00488	.28E-44	.591032	.973508	.005229	.95E-45	.45833	.026513	.88E-05	.62E-85	.977235	.947889	.44E-05	.77E-86
	.25	2 .627894	1 .009878	0 .573343	1 .76E-08	2 .563005	0 .980086	0 .539124	1 .24E-09	7 .093958	1 .019948	0 .331628	2 .42E-14	6 .747947	0 .960665	0 .293414	3 .62E-17
	.25	2 .625366	1 .009967	0 .05287	2 .29E-36	2 .560539	0 .97997	0 .052778	4 .25E-36	7 .104809	1 .02015	0 .003433	5 .03E-69	6 .758269	0 .960454	0 .003418	1 .74E-68
0		2 .655465	1 .010022	0 .017945	5 .04E-46	2 .589895	0 .980029	0 .018336	5 .83E-46	7 .26087	1 .020248	0 .000459	2 .53E-88	6 .906717	0 .960564	0 .000475	3 .39E-88
	.8	2 .625384	1 .00922	0 .006552	8 .6E-57	2 .560556	0 .98073	0 .006884	2 .85E-56	7 .093788	1 .018616	7 .2E-05	5 .9E-110	6 .747785	0 .961926	7 .82E-05	5 .1E-109
		2 .627925	1 .010191	0 .004399	5 E-68	2 .563035	0 .979815	0 .00465	3 .31E-68	7 .071204	1 .020593	3 .11E-05	2 .2E-132	6 .726302	0 .960146	3 .42E-05	6 .3E-133
	.25	2 .610392	1 .008113	0 .572347	5 .54E-10	2 .558699	0 .983901	0 .544233	1 .33E-11	6 .955362	1 .016365	0 .329786	2 .1E-16	6 .68262	0 .968132	0 .298398	1 .53E-20
0	.25	2 .6038	1 .007801	0 .051966	1 .18E-43	2 .552238	0 .984184	0 .051929	1 .21E-43	6 .923544	1 .015725	0 .003172	1 .04E-83	6 .65205	0 .968682	0 .00316	1 .4E-83
		2 .614278	1 .008319	0 .018059	8 .76E-59	2 .562508	0 .98369	0 .018377	1 .36E-58	6 .983848	1 .016773	0 .000434	7 .3E-114	6 .70999	0 .967713	0 .000448	1 .7E-113
		2	1	0	2	2	0	0	9	6	1	6	6	6	0	7	8

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	.8	.592646	.00776	.006537	.57E-75	.541305	.984179	.0068	.93E-76	.871834	.015637	.6E-05	.5E-147	.602368	.968667	.07E-05	.6E-148
		2 .602589	1 .007783	0 .004492	2 .29E-85	2 .551051	0 .9842	0 .004701	1 .18E-84	6 .915305	1 .015685	3 .16E-05	2 .9E-167	6 .644135	0 .968709	3 .43E-05	7 .2E-166

V. Conclusions

1. The estimated values for the Pareto distribution parameters, the reliability function, and the Reliability series system function, respectively, to five experiments, the non-parametric estimation (Empirical) methods was better than the MLE(maximum likelihood estimation) method.

2. When the sample size (n) is large, the estimated values for the Pareto distribution parameters, the reliability function, and the reliability function for the Reliability series system function, respectively, to five experiments, are reduced.

3. The MSE statistic values for the non-parametric method of (Empirical) are less than the MLE method and this indicates the advantage of the (Empirical) method.

4. MSE values decrease as the sample size (n) increases, for all estimated values, and for both methods.

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