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L (R) Cyclic Semigroups Satisfying the Identity: abc = ca

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Abstract--- *Semigroups being one of the algebraic structures are sets with associative binary operation defined on them. The theory of semigroups satisfy additional properties like commutative, Left (Right) cyclic i.e., L(R) cyclic , Left(Right) identity, Left(Right) cancellative and many others. In this paper we determine different structures of semigroups like normal, seminormal, quasinormal, semiregular and others by using the identity abc = cawith the concept of L(R) cyclic properties of semigroups.*

Keywords--- *Semigroup, Normal, Seminormal, Regular, Semiregular, Quasinormal.*

I. INTRODUCTION

Semigroup is an important algebraic structure with a binary operation satisfying closure and associative properties [1,2]. The main attempt of theory of semigroups is to generalize the concept of the groups. Also the study of the theory consists of algebraic abstraction of the properties of composition of transformation on a set [3].

From the past few decades the theory of semigroups had become a self-established branch of modern algebra linked strongly with different fields in Mathematics such as Group theory and Ring theory, Functional analysis and Differential geometry [4,5].

Applications of semigroups are of high interest that can be seen in Automata theory, Formal languages, Sociology, Biology and Biochemistry [6,7].

In the present work we use L (R) cyclic properties of semigroups with an identity $\boldsymbol{abc = ca}$and study different structures of semigroups involved [8].

1.1 Definition: A semigroup is a nonempty set S together with a binary operation '.' from $S\ X\ S \rightarrow S$. Thus we state the condition of (S,.) to be a semigroup as:

$$(a.b).c = a.(b.c) \quad or \quad (ab)c = a(bc) \quad for\ all a,b,c\ in\ S$$

1.2 Definition: If a semigroup (S,.) satisfies the identity $a(bc) = b(ca) = c(ab)$ for all a,b,c in S then S is said to be **L- cyclic.**

1.3 Definition: If a semigroup (S,.) satisfies the identity $(ab)c = (bc)a = (ca)b$ for all a,b,c in S then S is said to be **R- cyclic.**

1.4 Definition:If a semigroup (S, .) satisfies the identity $ab = ba$ for all a,b in S, then S is said to be **commutative.**

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1.5 Definition: If a semigroup (S, .) satisfies the identity $abca = acbca$ ($abca = abcba$) for all a, b, c in S then S is said to be **left (right) seminormal**.

1.6 Definition: If a semigroup (S, .) satisfies the identity $abca = acba$ for all a, b, c in S then S is **normal**.

1.7 Definition: A semigroup (S, .) is **left (right) semiregular** if it satisfies

$$abca = abacabca\ (abca = abcabaca)\ \forall\ a, b, c \in S.$$

1.8 Definition:.A semigroup (S, .) is **regular** if it satisfies $abca = abaca \forall\ a, b, c \in S$.

1.9 Definition: A **left(right) quasinorma**l semigroup (S,.) is a semigroup satisfying the identity $abc = acbc$ ($abc = abac$)$\forall a, b, c$ in S

1.10Definition: A semigroup (S,.) is said to **admit conjugates** if for all a, b∈ S there exists an element c∈S such that $ab = bc$ then **c is called conjugate of a by b** and is denoted by a^b .

1.11Definition: A **weakly separative** semigroup (S,.) is a semigroup satisfying the identity, $a^2 = ab = b^2 \Rightarrow a = b \forall a, b$ in S.

1.12Definition: A semigroup (S, .) is said to possess a **left (right) identity** if, for all a ∈ S there exists an element e ∈ S such that $ea = ae = a$.

Remark: In any semigroup of S, S is L- cyclic ⇔ S is R- cyclic.

Theorem 2.1: Let (S, .) be a L-cyclic semigroup where **"e"** is the identity. If (S,.) satisfies the identity$abc = ca$, $\boldsymbol{where\ a, b, c \in S}$ then**S admits conjugates**.

Proof: Consider the semigroup (S, .).

Then S**admitsconjugates** if for all $x, y \in S$ there exists an element $z \in S$such that $xy = yz$ then z is called conjugate of $\boldsymbol{x}$ by $\boldsymbol{y}$ and is denoted by x^y.

Now S is L- cyclic.

i.e., $x.(y.z) = y.(z.x) = z.(x.y)$for all $x, y, z \in$ S------- (1)

Now $x.(y.z) = xyz$

$\Rightarrow x.(y.e) = xye = xy$

Also $y.(z.x) = yzx$

$\Rightarrow y.(z.e) = yze = yz$

Also $z.(x.y) = zxy$

$\Rightarrow z.(x.e) = zxe = zx$

Thus from (1) $x.(y.z) = y.(z.x) = z.(x.y)$

implies $xy = yz = zx$

⇒S **admits conjugates.**

Theorem 2.2: Let (S, .) be a L-cyclic semigroup satisfying the identity$abc = ca$, $\boldsymbol{where\ a, b, c \in S}$ then **(S, .) is weakly separative.**

Proof: Let (S, .) be a semigroup.

Also S is L- cyclic.

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i.e., $a.(b.c) = b.(c.a) = c.(a.b)$for all $a, b, c \in$ S------- (1)

Now $a.(b.c) = ca$ [From the identity $abc = ca$]

Put $c = a$ then $a.(b.c) = a^2$

Also $b.(c.a) = ab$ [From the identity $abc = ca$]

Now $c.(a.b) = bc$[From the identity $abc = ca$]

Put $c = a$ then $c.(a.b) = ab$

Now $a.(b.c) = b.(c.a) = c.(a.b)$implies [From (1)]

$$a^2 = ab$$

⇒**S is weakly separative.**

Theorem 2.3: Let a L(R) cyclic semigroup (S,.) satisfies $\boldsymbol{abc = ca,\ where\ a, b, c \in S}$ then (S, .) is left (right) seminormal iff it is left(right) semiregular.

Proof: Let a L(R) cyclic semigroup (S,.) satisfies

$$\boldsymbol{abc = ca, where\ a, b, c \in S}$$

Case-1 :Consider S to be left seminormal.

$$abca = acbca$$

$= acb(ca)$ [associativity]

$= acb(abc)$ [$abc = ca$]

$= acba(bc)$ [associativity]

$= acba(cab)$ [$abc = ca$]

$= acba(bca)$[L(R)-cyclic]

$$= acbabca$$

$= a(cb)abca$ [associativity]

$= a(bac)abca$ [$abc = ca$]

$$\Rightarrow abca = abacabca$$

⇒ S is left semiregular.

Case-2:Now let S be left semiregular then,

$abca = abacabca$

$= aba(cab)ca$ [associativity]

$= aba(bc)ca$ [$abc = ca$]

$= ab(abc)ca$ [associativity]

$= ab(cab)ca$ [L(R)-cyclic]

$= (abc)abca$ [associativity]

$= ca(abca)$ [$abc = ca$]

$= (caa)bca$ [associativity]

$= (aca)bca$ [L(R)-cyclic]

$= a(cab)ca$ [associativity]

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$= a(bc)ca$ $[abc = ca]$

$= a(bcc)a$ [associativity]

$$\Rightarrow abca = acbca$$

$\Rightarrow$ S is left seminormal.

Theorem 2.4: Let a L(R) cyclic semigroup (S,.) satisfies $\boldsymbol{abc = ca, where\ a, b, c \in S}$ then (S, .) is left (right) semiregular, iffit is regular.

Proof: Consider a L(R) cyclic semigroup(S,.) satisfying

$$\boldsymbol{abc = ca, where\ a, b, c \in S}$$

Case-1: If S is regular, then

$$abca = abaca$$

$= a(ba)ca$ [associativity]

$= a(acb)ca$ $[abc = ca]$

$= a(bac)ca$ [L(R) -cyclic]

$= abac(ca)$ [associativity]

$= abac(abc)$ $[abc = ca]$

$= abaca(bc)$ [associativity]

$= abaca(cab)$ $[abc = ca]$

$= abaca(bca)$ [L(R) -cyclic]

$$\Rightarrow abca = abacabca$$

$\Rightarrow$ S is left semiregular.

Case-2: Let S be left semiregular, then

$$abca = abacabca$$

$= aba(cab)ca$ [associativity]

$= aba(bc)ca$ $[abc = ca]$

$= abab(cca)$ [associativity]

$= abab(ac)$ $[abc = ca]$

$= aba(bac)$ [associativity]

$= aba(cba)$ [L(R)-cyclic]

$= aba(ac)$ $[abc = ca]$

$= ab(aac)$ [associativity]

$= ab(aca)$ [L(R) -cyclic]

$$\Rightarrow abca = abaca$$

$\Rightarrow$ S is regular.

Theorem 2.5: Let a L(R) cyclic semigroup (S,.) satisfies $\boldsymbol{abc = ca, where\ a, b, c \in S}$ then (S, .) isleft (right) seminormal, iff it is normal.

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Proof: Consider L(R) cyclic semigroup 'S' that satisfies

$$\boldsymbol{abc = ca}, \quad \boldsymbol{where\ a, b, c \in S}$$

Case 1: If S is normal, then

$$abca = acba$$

$= (ac)\ ba$ [associativity]

$= (cba)\ ba$ $[abc = ca]$

$$= cbaba$$

$= (cb)(ab)a$ [associativity]

$= cb(bca)a$ $[abc = ca]$

$= cb(abc)a$ [L(R) -cyclic]

$= (cba)(bca)$ [associativity]

$= (acb)(abc)$ [L(R)-cyclic]

$= acb(ca)$ $[abc = ca]$

$$\Rightarrow abca = acbca$$

$\Rightarrow$ S is left (right) seminormal.

Case-2: Let S be left (right) seminormal, then

$$abca = acbca$$

$= (acb)ca$[associativity]

$= (ba)ca$ $[abc = ca]$

$= (bac)a$ [associativity]

$= (acb)a$ [L(R) -cyclic]

$$\Rightarrow abca = acba$$

$\Rightarrow$ S is normal.

Theorem 2.6: Let a L(R) cyclic semigroup (S,.) satisfies $\boldsymbol{abc = ca}$, $\boldsymbol{where\ a, b, c \in S}$ then (S,.) is left (right) semiregular iff it is right (left) semiregular.

Proof: Consider L(R) cyclic semigroup that satisfies

$$\boldsymbol{abc = ca}, \quad \boldsymbol{where\ a, b, c \in S}$$

Case-1 : If S is left semiregular, then

$$abca = abacabca$$

$= ab(aca)bca$ [associativity]

$= ab(caa)bca$ [L(R)-cyclic]

$= abca(abc)a$ [associativity]

$= abca(ca)a$ $[abc = ca]$

$= abca(caa)$ [associativity]

$= ab(ca)(aca)$ [L(R)-cyclic]

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$= ab(abc)(aca)$ [$abc = ca$]

$= ab(cab)aca$ [L(R)-cyclic]

$$\Rightarrow abca = abcabaca$$

$\Rightarrow$ S is right semiregular.

Case-2: Now let S be right semiregular, then

$$abca = abcabaca$$

$= ab(cab)aca$ [associativity]

$= ab(abc)aca$ [L(R)-cyclic]

$= aba(bca)ca$ [associativity]

$= aba(cab)ca$ [L(R)cyclic]

$= aba(cab)aca$ [$abc = ca$]

$= abacab(aca)$ [associativity]

$= abacab(aac)$ [L(R)-cyclic]

$$\Rightarrow abca = abacabca$$

$\Rightarrow$ S is left semiregular.

Theorem 2.7: Let a L(R) cyclic semigroup (S,.) satisfies $\boldsymbol{abc = ca, where\ a, b, c \in S}$ then (S, .) is left (right) seminormal if and only if it is right(left) seminormal.

Proof: Consider a L(R) cyclic semigroup 'S'that satisfies

$$\boldsymbol{abc = ca, where\ a, b, c \in S}$$

Case-1: If S is (left) seminormal, then

$$abca = acbca$$

$= a(cbc)a$ [associativity]

$= a(bcc)a$ [L(R)-cyclic]

$= ab(cca)$ [associativity]

$= ab(ac)$ [$abc = ca$]

$$\Rightarrow abca = abcba$$

$\Rightarrow$ S is right seminormal

Case-2: Now let S be right seminormal, then

$$abca = abcba$$

$= a(bcb)a$ [associativity]

$= a(cbb)a$ [L(R)-cyclic]

$= ac(bba)$ [associativity]

$= ac(ab)$ [$abc = ca$]

$$\Rightarrow abca = acbca$$

$\Rightarrow$S is left seminormal.

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Theorem 2.8: Let a L(R) cyclic semigroup(S,.) satisfies$\boldsymbol{abc = ca}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is normal iff it is right(left) seminormal.

Proof: ConsiderL(R) cyclic semigroup 'S' that satisfies

$$\boldsymbol{abc = ca}, \qquad \boldsymbol{where\ a, b, c \in S}$$

Case-1: If S is (left) seminormal, then

$$abca \iff acba$$

$\iff a(cb)a$ [associativity]

$\iff a(bac)a$ $[abc = ca]$

$\iff a(ba)ca$ [associativity]

$\iff a(acb)ca$ $[abc = ca]$

$\iff a(cba)ca$ [L(R)-cyclic]

$\iff acb(aca)$[associativity]

$\iff acb(aac)$[L(R)-cyclic]

$$\Rightarrow abca = acb(ca)$$

$\Rightarrow$ S is left seminormal

Case-2: Now again if S is normal, then

$$abca = acba$$

$\iff a(cb)a$ [associativity]

$\iff a(bac)a$ $[abc = ca]$

$\iff ab(aca)$[associativity]

$\iff ab(caa)$[L(R)-cyclic]

$\iff abc(aa)$[associativity]

$\iff abc(aba)$$[abc = ca]$

$\iff abc(aab)$[L(R)-cyclic]

$$\Rightarrow abca = abcba$$

$\Rightarrow$ S is right seminormal.

Theorem 2.9: Let a L(R) cyclic semigroup(S,.) satisfies$\boldsymbol{abc = ca}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is normal iff it is left(right) quasi normal.

Proof: Consider a L(R) cyclic semigroup that satisfies

$$\boldsymbol{abc = ca}, \qquad \boldsymbol{where\ a, b, c \in S}$$

Case-1 : If S is (left) quasi normal, then

$$abc = acbc$$

$$\Rightarrow abca = (acbc)a$$

$= ac(bca)$[associativity]

$= ac(ab)$$[abc = ca]$

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$= a(ca)b$ [associativity]

$= a(abc)b$ $[abc = ca]$

$= a(cab)b$ [L(R)-cyclic]

$= ac(abb)$[associativity]

$$\Rightarrow abca = acba$$

⇒ S isnormal.

Case-2:Now let S be normal, then

$$abca = acba$$

$$\Rightarrow abcab = acbab$$

$\Rightarrow ab(cab) = (acb)(ab)$[associativity]

$\Rightarrow ab(bc) = (cba)(bca)$[L(R)-cyclic] and $[abc = ca]$

$\Rightarrow a(bbc) = (ac)(cab)$ [L(R)-cyclic] and $[abc = ca]$

$\Rightarrow a(cbb) = (ac)(bc)[abc = ca]$

$$\Rightarrow abc = acbc$$

⇒ S is left quasinormal.

Theorem 2.10: Let a L(R)cyclic semigroup(S,.) satisfies$\boldsymbol{abc = ca}$, $\boldsymbol{where\ a, b, c \in S}$ then (S, .) is regular iff it is left (right) quasi normal.

Proof: Consider L(R) cyclic semigroup that satisfies

$$\boldsymbol{abc = ca, where\ a, b, c \in S}$$

Case-1: If S is left quasi normal, then

$$abc = acbc$$

$$\Rightarrow abca = acbca$$

$= a(cb)ca$ [associativity]

$= a(bac)ca$ $[abc = ca]$

$= (aba)(cca)$[associativity]

$= aba(acc)$[L(R)-cyclic]

$= aba(ca)[abc = ca]$

$$\Rightarrow abca = abaca$$

⇒ S is regular.

Case-2: Now let S be regular semigroup, then

$$abca = abaca$$

$$\Rightarrow abcab = abacab$$

$\Rightarrow ab(cab) = a(ba)(cab)$[associativity]

$\Rightarrow ab(bc) = a(acb)(bc)[abc = ca]$

$\Rightarrow a(bbc) = (aac)(bbc)[abc = ca]$

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$\Rightarrow a(cbb) = (caa)(cbb)$ [L(R)-cyclic]

$$\Rightarrow abc = acbc$$

$\Rightarrow$ S is left quasi normal.

II. CONCLUSION

The present paper mainly focuses on different structures of semigroups which are studied through L (R) cyclic properties with an identity $abc = ca$. The work can also be extended with different properties and stuctures of semigroups.

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