International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 05, 2020
ISSN: 1475-7192

Applications of A Shehu Transform to the Heat and Transport Equations

Athraa Neamah ALbukhuttar*
Zainab Dheyaa Ridha and Hayder Neamah Kadhim

Abstract--- *The Shehu transform principle properties were showed by Maitama [13]. Atheros in 2019 used Shehu transform to solve differential equations. In this paper, we introduce Shehu transform, which used in solution of ordinary and partial differential equations, moreover, we extended Shehu transform application for solving transport and heat equations that satisfied known or unknown initial conditions*.

Keywords--- *Transport Equations, Applications of A Shehu, Uniqueness.*

I. INTRODUCTION

Transport and Heat equations are partial differential equations which the applications wide in physics and engineering [6].

Moreover, integral transformations are mathematical methods that extensively used in solution of differential equations, therefore, there are several integral transforms such as the Laplace, Sumudu, Mellin, Elzaki and Temimi [3,4,5,10], to name but a few.

Laplace transform is widely used to solve differential equations subjected to the initial or boundary conditions.

The result of solutions for initial value problems that used Laplace transform represent particular solutions. Sumudu transform is similar to Laplace transform, but the first used to solve differential equations with variable coefficient [5].

In 2013, Atangana and Kilicman introduced a new integral transform named the Abdon - Kilicman integral transform for solution some differential equations with some kind of Uniqueness [1]. The new integral transform is defined as:

$$M_n(s) = M_n[f(x)](s) = \int_0^\infty x^n \exp(-xs)\, f(x)dx, \; (1.1)$$

The Atangana - Kilicman integral happened Laplace transform when *n=0*.

A new integral transform named the $\mathbb{M}$- transform, which is Moreover similar to normal transform is introduced by Srivastava et al. [9] in 2015. Mathematically talking - transform is Closely related with the known Laplace transform and the Sumudu integral transform. $\mathbb{M}$- transform was successfully used to first order initial-boundary value problem. The $\mathbb{M}$- transform is defined as:

Athraa Neamah ALbukhuttar, Department of Mathematics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq.*
E-mail: athraan.kadhim@uokufa.edu.iq
Zainab Dheyaa Ridha, Department of Banking & Financial, Faculty of Administration and Economics, University of Kufa, Najaf, Iraq.
Hayder Neamah Kadhim, Department of Banking & Financial, Faculty of Administration and Economics, University of Kufa, Najaf, Iraq.
E-mail: hayder.najaf1976@gmail.com

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$\mathbb{M}_{p,m}[f(t)](u,v) = \int_0^\infty \frac{\exp(-ut)f(vt)}{(t^m+v^m)^p}\,dt, (1.2)$$

$p \in \mathbb{c}$; $\Re(p), \geq 0, m \in \mathbb{z}_+ = 1,2,3..$, where both $u \in \mathbb{c}$ and $v \in \mathbb{R}_+$ are the $\mathbb{M}-$ transform variables.

Recently, a Laplace - kind integral transform named the Yang transform for solution fixed heat transfer problems was submitted in 2016[15]. The integral transform is defined as:

$$Y[\emptyset(t)] = \emptyset(w) = \int_0^\infty \exp\left(\frac{-t}{w}\right)\emptyset(t)dt \ (1.3)$$

Under condition the integral convergent.

Because the quick development in the physical science and engineering models, there are a lot of another integral transforms in the literature. Anyway, most of the present integral transforms have some determinants and cannot be applied immediately to solved nonlinear problems or many composite mathematical models. As a result, many authors exists very interested to reach with the replacement approach for solution a lot of genuine - life problems.

In 2016, [2] Atangana and Alkaltani introduced a Novel double integral transform and their properties according to on the Laplace transform and decomposition method. The double integral transform was successfully used to second order partial differential equation with uniqueness named the two dimensional Mboctara equation. Recently, Eltayeb used double Laplace decomposition method to Mon linear partial differential equations [8]. In 2017, Bel-gacem el at, extended the applications of the normal and the Sumudu transforms to fractional diffusion and Stokes fluid flow realms [7].

We benefited by the above - mentioned researches, in this paper we Recommended Laplace - type integral transform named Shehu transform for solution both ordinary and partial differential equations. The Laplace - kind integral transform converges to Laplace transform when k=1, and to Yang integral transform when ƀ=1 the recommended integral transform is successfully used to solve many types of differential equations [11, 12, 14]. All the results it got in the applications section can easily be verification applying the Laplace or Fourier integral transforms. In this paper, the Shehu transform is denoted by an operator $\mathbb{s}$.

In our research, we apply Shehu transform to solve transport and heat equations in one - dimension. Whereas, they are homogeneous or non- homogeneous, that subjected to known or unknown initial conditions. Section 2 showed the properties and theorems for Shehu transform, and the most important formulas of Shehu transform for some functions. In section 3, we established the general formula to the transport and heat equations, in one – dimension by using Shehu transform. Finally, section 4 applied the general formula for equations in some examples.

II. Basic Definitions And Properties

Definition: The Shehu transform of the function $\mathrm{u(t)}$ of exponential order is over the set of functions

$$\mathrm{A} = \{\mathrm{u(t)}: \exists\ \mathrm{N}, \eta_1, \eta_2 > 0, |\mathrm{u(t)}| < \mathrm{N}\exp(\frac{|\mathrm{t}|}{\eta_i}), \mathrm{if}\ \mathrm{t} \in (-1)^{\mathrm{i}} \times [0,\infty)\}$$

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

By the following integral:

$$\mathbb{s}[\mathrm{u(t)}] = \mathrm{U}(\mathfrak{b}, \kappa) = \int_0^\infty \exp\left(\frac{-\mathfrak{b}\mathrm{t}}{\kappa}\right) \mathrm{u(t)dt}$$

$$= \lim_{\alpha\to\infty} \int_0^\alpha \exp\left(\frac{-\mathfrak{b}\mathrm{t}}{\kappa}\right) \mathrm{u(t)dt}\,;\ \mathfrak{b} > 0, \kappa > 0 \quad (2.1)$$

It converges if the limit of the integral exists, and diverges if not.

The inverse Shehu transform is given by:

$$\mathbb{s}^{-1}[\mathrm{U}(\mathfrak{b}, \kappa)] = \mathrm{u(t)}, \text{for } \mathrm{t} \geq 0 \quad (2.2)$$

Equivalently

$$\mathrm{u(t)} = \mathbb{s}^{-1}[\mathrm{U}(\mathfrak{b}, \kappa)] = \frac{1}{2\pi \mathrm{i}} \int_{\alpha - \mathrm{i}\infty}^{\alpha + \mathrm{i}\infty} \frac{1}{\kappa} \exp\left(\frac{\mathfrak{b}\mathrm{t}}{\kappa}\right) \mathrm{U}(\mathfrak{b}, \kappa)\, \mathrm{d}\mathfrak{b} \quad (2.3)$$

Where $\mathfrak{b}$ and κ are the Shehu transform variables, and α is a real constant and the integral in equation (2.3) is taken along $\mathfrak{b} = \alpha$ in the complex plane $\mathfrak{b} = \mathrm{x} + \mathrm{iy}$.

Property: Linearity property of Shehu *transform.*

Let the functions $\alpha\mathrm{u(t)}$ and $\beta w(t)$ be in set A,

then $\left(\alpha\mathrm{u(t)} + \beta w(t)\right) \in A, where\ \alpha\ and\ \beta\ are\ nonzero$ arbitrary constant and

$$\mathbb{s}[\alpha\mathrm{u(t)} + \beta\mathrm{w(t)}] = \alpha\mathbb{s}[u(t)] + \beta\mathbb{s}[\mathrm{w(t)}]. \quad (2.4)$$

Proof. Using the *definition* of Shehu transform, we get:

$$\mathbb{s}[\alpha\mathrm{u(t)} + \beta\mathrm{w(t)}] = \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \left(\alpha\mathrm{u(t)} + \beta\mathrm{w(t)}\right) dt$$

$$= \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \left(\alpha\mathrm{u(t)}\right) dt + \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \left(\beta\mathrm{w(t)}\right) dt\,.$$

$$= \alpha \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \mathrm{u(t)} dt + \beta \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \mathrm{w(t)} dt\,.$$

$$= \alpha u \int_0^\infty exp(-\mathfrak{b}t) \mathrm{u}(\kappa\mathrm{t}) dt + \beta\kappa \int_0^\infty exp(-\mathfrak{b}t) \mathrm{w}(\kappa\mathrm{t}) dt\,.$$

$$= \alpha\mathbb{s}[u(t)] + \beta\mathbb{s}[\mathrm{w(t)}].$$

Lemma: *Derivative* of Shehu transform.

If the function $u^{(n)}(t)$ is the $n-th$ derivative of the function $u(t) \in A$ with respect to t then its Shehu transform is defined by:

$$\mathbb{s}[u^n(t)] = \frac{\mathfrak{b}^n}{\kappa^n} V(\mathfrak{b}, \kappa) - \sum_{k=0}^{n-1} \left(\frac{\mathfrak{b}}{\kappa}\right)^{n-(k+1)} u^k(0). \quad (2.5)$$

When *n*=1, 2, and 3 in equation (2.5) above, we obtain the following *derivatives* with respect to t

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$\mathbb{s}[u'(t)] = \frac{\mathfrak{b}}{\kappa} U(\mathfrak{b},\kappa) - u(0) \quad (2.6)$$

$$\mathbb{s}[u''(t)] = \frac{\mathfrak{b}^2}{\kappa^2} U(\mathfrak{b},\kappa) - \frac{\mathfrak{b}}{\kappa} u(0). \quad (2.7)$$

$$\mathbb{s}[u'''(t)] = \frac{\mathfrak{b}^3}{\kappa^3} U(\mathfrak{b},\kappa) - \frac{\mathfrak{b}^2}{\kappa^2} u(0) - \frac{\mathfrak{b}}{\kappa} u'(0) - u''(0) \quad (2.8)$$

Proof. Now suppose equation (2.5) is true for $n = k$ then using equation (2.8) and the *induction* hypothesis, we deduce

$$\mathbb{s}\left[\left(u^{(k)}(t)\right)'\right] = \frac{\mathfrak{b}}{\kappa} \mathbb{s}[u^k(t)] - u^k(0).$$

$$= \frac{\kappa}{\mathfrak{b}} \left[\frac{\mathfrak{b}^k}{\kappa^k} \mathbb{s}[u(t)] - \sum_{i=o}^{k-1} \left(\frac{\mathfrak{b}}{\kappa}\right)^{k-(i+1)} u^{(i)}(0) \right] - u^{(k)}(0).$$

$$= \left(\frac{\mathfrak{b}}{\kappa}\right)^{k+1} \mathbb{s}[u(t)] - \sum_{i=0}^{k} \left(\frac{\mathfrak{b}}{\kappa}\right)^{k-i} u^i(0)$$

Which implies that equation (2.8) holds for $n = k + 1$, by induction hypothesis the proof is complete.

The following important properties are obtain *using* the Leibniz's rule

$$\mathbb{s}\left[\frac{\partial u(x,t)}{\partial x}\right] = \int_0^\infty \exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \frac{\partial u(x,t)}{\partial x} dt = \frac{\partial}{\partial x} \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) u(x,t) dt = \frac{\partial}{\partial x}[U(x,\mathfrak{b},\kappa)] = \frac{d}{dx}[U(x,\mathfrak{b},\kappa)].$$

$$\mathbb{s}\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = \int_0^\infty \exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \frac{\partial^2 u(x,t)}{\partial x^2} dt = \frac{\partial^2}{\partial x^2} \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) u(x,t) dt = \frac{\partial^2}{\partial x^2}[U(x,\mathfrak{b},\kappa)] = \frac{d^2}{dx^2}[U(x,\mathfrak{b},\kappa)]$$

.

.

.

$$\mathbb{s}\left[\frac{\partial^n u(x,t)}{\partial x^n}\right] = \int_0^\infty \exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) \frac{\partial^n u(x,t)}{\partial x^n} dt = \frac{\partial^n}{\partial x^n} \int_0^\infty exp\left(\frac{-\mathfrak{b}t}{\kappa}\right) u(x,t) dt = \frac{\partial^n}{\partial x^n}[U(x,\mathfrak{b},\kappa)]$$

$$= \frac{d^n}{dx^n}[U(x,\mathfrak{b},\kappa)]$$

Table 1: The following table showed the Shehu Transform for some functions such as:

S.No.	$u(t)$	$\mathbb{s}\,[u(t)]$
1	1	$\frac{\kappa}{\mathfrak{b}}$
2	t	$\frac{\kappa^2}{\mathfrak{b}^2}$

3	$\exp(\propto (t))$	$\frac{\kappa}{\mathfrak{b}-\propto \kappa}$
4	$\sin(\propto t)$	$\frac{\propto \kappa^2}{\mathfrak{b}^2 +\propto^2 \kappa^2}$
5	$\cos(\propto t)$	$\frac{\kappa \mathfrak{b}}{\mathfrak{b}^2 +\propto^2 \kappa^2}$
6	$t \exp(\propto t)$	$\frac{\kappa^2}{(\mathfrak{b}-\propto \kappa)^2}$
7	$\frac{\exp(\beta t)\sin(\propto t)}{\propto}$	$\frac{\kappa^2}{(\mathfrak{b}-\beta \kappa)^2 +\propto^2 \kappa^2}$
8	$\frac{\exp(\propto t)}{\beta-\propto}$	$\frac{\kappa^2}{(\mathfrak{b}-\propto \kappa)(\mathfrak{b}-\beta \kappa)}$

III. FORMULA OF GENERAL SOLUTIONS OF HEAT AND TRANSPORT EQUATIONS

Formula 1: Consider the *homogeneous* transport equations:

$$u_t + BDu = 0 \; (3.1)$$

$$u(x,0) = g(x)$$

We take the Shehu *transformation* to both sides, we get:

$$\frac{\mathfrak{b}}{\kappa} U(x,\mathfrak{b},\kappa) - U(x,0) - B\frac{dU(x,\mathfrak{b},\kappa)}{dx} = 0$$

By *substitute*, initial value:

$$\frac{\mathfrak{b}}{\kappa} U(x,\mathfrak{b},\kappa) - g(x) + B\frac{dU(x,\mathfrak{b},\kappa)}{dx} = 0$$

$$\grave{U} + \frac{\mathfrak{b}}{B\kappa} U = \frac{1}{B} g(x) \; (3.2)$$

Which represent linear equations of *order* one and it has the solution

$$U = \frac{1}{B} e^{\frac{-\mathfrak{b}}{B\kappa}x} \int_a^x g(\theta).e^{\frac{\mathfrak{b}}{B\kappa}\theta} d\theta$$

We take *the* inverse of both sides:

$$u(x,t) = \frac{1}{B} \mathbb{S}^{-1}\left[e^{\frac{-\mathfrak{b}}{B\kappa}x} \int_a^x g(\theta).e^{\frac{\mathfrak{b}}{B\kappa}\theta} d\theta \right], (3.3)$$

Equation (3.3) represent the general *formula* solution of equation (3.1)

Formula 2: Consider the non- *homogeneous* transport equations:

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$u_t + BDu = f(x,t)\ (3.4)$$

$u(x,0) = g(x)$

After we take the Shehu *transformation* to both sides, we have:

$$\frac{\mathfrak{b}}{\kappa}U(x,\mathfrak{b},\kappa) - U(x,0) + B\frac{dU(x,\mathfrak{b},\kappa)}{dx} = \mathbb{s}[f(x,t)]$$

By *substitute*, initial value:

$$B\frac{dU(x,\mathfrak{b},\kappa)}{dx} + \frac{\mathfrak{b}}{\kappa}U(x,\mathfrak{b},\kappa) = g(x) + \mathbb{s}[f(x,t)]$$

$$\frac{dU(x,\mathfrak{b},\kappa)}{dx} + \frac{\mathfrak{b}}{B\kappa}U(x,\mathfrak{b},\kappa) = \frac{1}{B}(\mathbb{s}[f(x,t) + g(x)])$$

The above *equation* has the solution

$$U = \frac{1}{B}e^{\frac{-\mathfrak{b}}{B\kappa}x}\int_a^x \mathbb{s}[f(\theta,t)].e^{\frac{\mathfrak{b}}{B\kappa}\theta}d\theta + \frac{1}{B}e^{\frac{-\mathfrak{b}}{B\kappa}x}\int_a^x g(\theta).e^{\frac{\mathfrak{b}}{B\kappa}\theta}d\theta\ (3.5)$$

The general formula solution of equation (3.4) can be obtained by taking the *inverse* of both sides to equation (3.5)

$$u(x,t) = \frac{1}{B}\mathbb{s}^{-1}\left[e^{\frac{-\mathfrak{b}}{B\kappa}x}\int_a^x \mathbb{s}[f(\theta,t)].e^{\frac{\mathfrak{b}}{B\kappa}\theta}d\theta\right] + \frac{1}{B}\mathbb{s}^{-1}\left[e^{\frac{-\mathfrak{b}}{B\kappa}x}\int_a^x g(\theta).e^{\frac{\mathfrak{b}}{B\kappa}\theta}d\theta\right]\ (3.6)$$

Formula 3: Consider *the* homogeneous heat equations:

$$u_t - Bu_{xx} = 0\ (3.7)$$

$$u(x,0) = g(x)$$

We take the Shehu transformation to both *sides* and substitute the initial value:

$$B\frac{d^2U(x,\mathfrak{b},\kappa)}{dx^2} - \frac{\mathfrak{b}}{\kappa}U(x,\mathfrak{b},\kappa) = -g(x)$$

The last equation represent linear equation of *second* order, which has the solution:

$$U(x,\mathfrak{b},\kappa) = c_1e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} + c_2e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} - \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x g(\theta).e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}d\theta\right].e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}$$

$$+\frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x g(\theta).e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}d\theta\right].e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}\ (3.8)$$

We take the inverse of both sides to equation (3.8), we get:

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$u(x,t) = \mathbb{s}^{-1}\left[c_1 e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} + c_2 e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} - \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x g(\theta).\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}\right.$$

$$\left. + \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x g(\theta).\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}\right] \quad (3.9)$$

Formula 4: Consider *the* non- homogeneous heat equations:

$$u_t - Bu_{xx} = f(x,t) \quad (3.10)$$

$$u(x,0) = 0$$

After taking the Shehu transformation and applying the initial *conditions* of equation (3.10), we obtained:

$$B\frac{d^2U(x,\mathfrak{b},\kappa)}{dx^2} - \frac{\mathfrak{b}}{\kappa}U(x,\mathfrak{b},\kappa) = -\mathbb{s}[f(x,t)],$$

Which represent linear *equation* of second order and it has the solution

$$U(x,\mathfrak{b},\kappa) = c_1 e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} + c_2 e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} - \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x \mathbb{s}[f(\theta,t)].\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}$$

$$+ \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x \mathbb{s}[f(\theta,t)].\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} \quad (3.11)$$

The solution of equation (3.10) can be found by *taking* the inverse of both sides to equation (3.11):

$$u(x,t) = \mathbb{s}^{-1}\left[c_1 e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} + c_2 e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x} - \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x \mathbb{s}[f(\theta,t)].\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}\right.$$

$$\left. + \frac{1}{2\sqrt{\frac{\mathfrak{b}}{B\kappa}}}\left[\int_a^x \mathbb{s}[f(\theta,t)].\, e^{\sqrt{\frac{\mathfrak{b}}{B\kappa}}\theta}\, d\theta\right].\, e^{-\sqrt{\frac{\mathfrak{b}}{B\kappa}}x}\right] \quad (3.12)$$

Equation (3.12) represent the general formula solution of equation (3.10).

IV. APPLICATIONS

In the following section, the usefulness and the effectiveness of Shehu transformation are showed by finding exact solution of transport and heat equations.

Example 1: To solve the equation

$$u_x + 3\, u_t = 0$$

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$u(x,0) = e^{x}$$

Sol: We can obtained the solution by utilizing of formula 1 in equation (3.3), as show:

$$u(x,t) = 3\mathbb{s}^{-1}\left[e^{\frac{-3b}{\kappa}x}\int_{a}^{x}.e^{\frac{\kappa+3b}{\kappa}\theta}d\theta\right]$$

$$u(x,t) = 3\mathbb{s}^{-1}\left[(\frac{\kappa}{3b+\kappa})e^{\frac{-3b}{\kappa}x}.e^{\frac{u+3b}{\kappa}x}\right]$$

$$= 3\mathbb{s}^{-1}\left[(\frac{\kappa}{3b+\kappa})e^{x}\right]$$

$$= e^{x}\exp(\frac{-1}{3}t)$$

Example 2: For solving the equation

$$3u_x + 4\,u_t = 2$$

$$u(x,0) = e^{3x}$$

Sol: By utilizing the formula 2 in equation (3.6), *we* obtained:

$$u(x,t) = \frac{4}{3}\mathbb{s}^{-1}\left[(\frac{1}{2})e^{\frac{-4b}{3\kappa}x}\int_{a}^{x}\frac{\kappa}{b}.e^{\frac{4b}{3\kappa}\theta}d\theta\right] + \frac{4}{3}\mathbb{s}^{-1}\left[e^{\frac{-4b}{3\kappa}x}\int_{a}^{x}e^{3\theta}.e^{\frac{4b}{3\kappa}\theta}d\theta\right]$$

$$u(x,t) = \frac{4}{3}\mathbb{s}^{-1}\left[(\frac{3}{8})(\frac{\kappa^2}{b^2})\right] + \frac{4}{3}\mathbb{s}^{-1}\left[\left(\frac{3\kappa}{4b+9\kappa}\right).e^{3x}\right]$$

$$= \frac{1}{2}\mathbb{s}^{-1}\left[\frac{\kappa^2}{b^2}\right] + \frac{4}{3}\mathbb{s}^{-1}\left[\left(\frac{3\kappa}{4b+9\kappa}\right).e^{3x}\right]$$

$$= \frac{1}{2}t + \frac{4}{3}e^{3x}\exp(\frac{-9}{4}t)$$

Example 3: To solve the equation

$$u_t = u_{xx}$$

$$u(x,0) = e^{\frac{1}{2}x}$$

Sol: We can use the formula 3 in *equation* (3.9), so as:

$$u(x,t) = \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{b}{\kappa}}}\left[\int_{a}^{x}e^{\frac{1}{2}\theta}e^{-\sqrt{\frac{b}{\kappa}}\theta}d\theta\right]e^{\sqrt{\frac{b}{\kappa}}x} + \frac{1}{2\sqrt{\frac{b}{\kappa}}}\left[\int_{a}^{x}e^{\frac{1}{2}\theta}\,e^{\sqrt{\frac{b}{\kappa}}\theta}d\theta\right]e^{-\sqrt{\frac{b}{\kappa}}x}\right]$$

$$u(x,t) = \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{b}{\kappa}}}\left[\int_{a}^{x}e^{(\frac{1}{2}-\sqrt{\frac{b}{\kappa}})\,\theta}d\theta\right]e^{\sqrt{\frac{b}{\kappa}}x} + \frac{1}{2\sqrt{\frac{b}{\kappa}}}\left[\int_{a}^{x}e^{(\frac{1}{2}+\sqrt{\frac{b}{\kappa}})\,\theta}\,d\theta\right]e^{-\sqrt{\frac{b}{\kappa}}x}\right]$$

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

$$u(x,t) = \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{\text{ƀ}}{\kappa}}}\left(\frac{1}{(\frac{1}{2}-\sqrt{\frac{\text{ƀ}}{\kappa}})}\right)e^{\frac{1}{2}x} + \frac{1}{2\sqrt{\frac{\text{ƀ}}{\kappa}}}\left(\frac{1}{(\frac{1}{2}+\sqrt{\frac{\text{ƀ}}{\kappa}})}\right)e^{\frac{1}{2}x}\right]$$

$$u(x,t) = \mathbb{s}^{-1}\left[e^{\frac{1}{2}x}\left(\frac{-1}{(\sqrt{\frac{\text{ƀ}}{\kappa}}-\frac{2\text{ƀ}}{\kappa})} + \frac{1}{(\sqrt{\frac{\text{ƀ}}{\kappa}}+\frac{2\text{ƀ}}{\kappa})}\right)\right]$$

$$= \mathbb{s}^{-1}\left[e^{\frac{1}{2}x}.\frac{\kappa}{(\text{ƀ}-\frac{1}{4}\kappa)}\right]$$

$$= e^{\frac{1}{2}x}.\exp(\frac{1}{4}t)$$

Example 4: To solve the non-*homogenous* heat equation

$$u_t + u_{xx} = e^{3x+t}$$

$$u(x,0) = e^{2x}$$

Sol.: Similarly, if we use the formula 4 in equation (3.12), then also the above *equations* has the solution:

$$u(x,t) = \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\int_a^x \mathbb{s}[e^{3\theta+t}]e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}x} + \frac{1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\int_a^x \mathbb{s}[e^{3\theta+t}]e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}x}\right]$$

$$= \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\int_a^x e^{3\theta}\left(\frac{\kappa}{\text{ƀ}-\kappa}\right)e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}x} + \frac{1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\int_a^x e^{3\theta}\left(\frac{\kappa}{\text{ƀ}-\kappa}\right)e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}x}\right]$$

$$= \mathbb{s}^{-1}\left[\frac{-1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\left(\frac{\kappa}{\text{ƀ}-\kappa}\right)\int_a^x e^{3\theta}e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}x} + \frac{1}{2\sqrt{\frac{-\text{ƀ}}{\kappa}}}\left[\left(\frac{\kappa}{\text{ƀ}-\kappa}\right)\int_a^x e^{3\theta}e^{\sqrt{\frac{-\text{ƀ}}{\kappa}}\theta}d\theta\right]e^{-\sqrt{\frac{-\text{ƀ}}{\kappa}}x}\right]$$

$$= \mathbb{s}^{-1}\left[e^{3x}(\frac{\kappa}{\text{ƀ}-\kappa})(\frac{-\kappa}{\text{ƀ}+9\kappa})\right]$$

$$= \frac{1}{10}e^{3x+t}$$

REFERENCES

[1] A Atangana and A Kilicman, A novel integral operator transform and its application to some FODE and FPDE with some kind of singularities, *Mathematical Problems in Engineering*, Special I ssue, 2013.

[2] A Atangana and B Alkaltani, A novel double integral transform and its applications, *Jour-nal of Nonlinear Science and Applications*, Vol.9(2), 2016.

[3] A H Mohammed and A N Kathem, Solving Euler's Equation by Using New Transformation, *Journal kerbala University*, Vol.6(4), 2008.

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020

[4] A N Albukhuttar and I H Jaber, Elzaki transformation of Linear Equation without Subject to any initial conditions, *Journal of Advanced Research in Dynamical and control Systems*, Vol.11(2), 2019.

[5] A N Kathem, On Solutions of Differential Equations by using Laplace Transformation, *The Islamic College University Journal*, Vol.7(1), 2008.

[6] B Goodwine, Engineering Differential Equations: Theory and Applications, Springer, *New York, USA*, 2010.

[7] F Belgacem and el at, New and extended applications of the natural and Sumudu transforms: Fractional diffusion and Stokes fluid flow realms, *Advances Real and Complex Analysis with Applications, Springer*, Singapore, 2017.

[8] H Eltayeb, A note on double Laplace decomposition method and nonlinear partial differential equations, New Trends in Mathematical Sciences, Vol.5(4), 2017.

[9] H M Srivastava, M Luo and R K Raina, A new integral transform and its applications, Acta Mathematical Scientia, Vol.35(6), 2015.

[10] L Boyadjiev and Y Luchko, Mellin Integral Transform Approach to Analyze the Multi-dimensional Diffusion-Wave Equations, *Chaos Solutions and Fractals*, Vol.102(1), 2017.

[11] S Aggarwal, A Gupta and S Sharma, A New Application of Shehu Transformation for Handling Volterra Integral Equations of first kind, *International Journal of Research in Advent Technology*, Vol.7(4), 2019.

[12] S Aggarwal, S D Sharma and A R Gupta, Application of Shehu Transformation Handling growth and decay problems, *Global Journal of Engineering Science and Researches*, Vol.6(4), 2019.

[13] S Maitama and W Zhao, New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations, *International Journal of Analysis and Applications*, Vol.17(2), 2019.

[14] S Maitama and W Zhao, New Laplace –type integral Transform for solving steady heat Transfer-problem, *Ther-mal Science*, Vol.2(4), 2019.

[15] X J Yang, A new integral transform method for solving steady heat-transfer problem, *Ther-mal Science*, Vol.20 (2), 2016.

DOI: 10.37200/IJPR/V24I5/PR2020141
Received: 08 Mar 2020 | Revised: 26 Mar 2020 | Accepted: 04 Apr 2020