Theoretical Research of Properties of Paraboloid surfaces with the Purpose of Substantiation of the Optimal Method of Adjustment of the Facets of the Concentrator

Shukhrat Avchiev, Juraeva Khusnora, Dilafruzkhon Tagaeva,
Maknona Xamidova and Akmaljon Yuldashev

Abstract--- *This article analyzes the properties of the relative positions of normals to a paraboloid surface. Theoretically, the existence of a region of minimum sizes in which all normals and a paraboloid intersect is found, and formulas for calculating the size and location of this region are obtained.*

Keywords--- *Properties of Paraboloidal Surfaces, Alignment of the Concentrator Facet, Instrument Defining the Normal, Normal Properties, Canonical Equation of a Parabola, Evolute, Tangent to an Evolute, Paraboloid of Rotation, Normal Angle Coefficient, Region with Minimum Dimensions, Paraboloidal Surface of the Concentrator, Orientation of Normals, Normals Intersection to Parabola by Opening the Parabola*.

I. INTRODUCTION

The main purpose of a solar high-temperature furnace is to collect a certain amount of solar energy and concentrate it on a small area. Concentration in this case is carried out by focusing the streams of sunlight.

Concentrators are powered by paraboloidal lattice, parabolotylindricated discharge and drug optimization systems.

The final characteristics of solar high-temperature furnaces depend on the quality of the formation of the reflecting surface of the concentrator and its changes during operation. As you know, the reflecting surface of the concentrator is formed from individual spherical facets, each of which has its own deviations from the design. Therefore, after installation and during the operation of the concentrator, it is required to perform alignment-alignment of the individual facets of the reflecting surface to the required spatial position on the supporting frame and orientation of the normals to the center of each facet.

II. FIELD OF STUDY

The study relates to solar engineering and can be used to assemble surfaces with a reflective coating as applied to focusing heliostats and solar energy concentrators.

Shukhrat Avchiev, Professor at Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan.
E-mail: saidov2030@yandex.com
Juraeva Khusnora, Senior lecturer Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan.
Dilafruzkhon Tagaeva, Senior lecturer Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan.
Maknona Xamidova, Associated Professor Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan.
Akmaljon Yuldashev, Senior lecturer Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan.

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III. INFORMATION ANALYSIS

One of the main disadvantages of a number of methods for adjusting the facet of the concentrator is the need for a significant number of installations of the device that sets the normal [1-9]. In order to eliminate this drawback, we analyze the properties of the relative position of the normals to the ideal paraboloid surface or its axial section (parabola).

To do this, using the canonical parabola equation

$$y = \frac{x^2}{2P}, \tag{1}$$

we obtain, according to well-known formulas [14,15], the coordinates of the center of curvature x_c, y_c of any point C (x, y) of the parabola, which at the same time are the parametric equations of the evolute.

$$\left.\begin{aligned} x_c &= x - \frac{y'\left(1+y'^2\right)}{y"} = x - \frac{\frac{x}{P}\left(1+\frac{x^2}{P^2}\right)}{1/P} \\ y_c &= y + \frac{1+y'^2}{y"} = y + \frac{1+\frac{x^2}{P^2}}{1/P} = -\frac{x^3}{P^2} \end{aligned}\right\} \tag{2}$$

Denoting the current coordinates of the centers of curvature by $X = x_c$, and expressing $Y = y_c$ must and expressing Y through X, we find the equation of parabola evolute

$$Y_E = P + \frac{3}{2}\sqrt[3]{X^2 \cdot P}. \tag{3}$$

Imagine that a paraboloid of rotation has specific dimensions, maybe opening and depth. This is equivalent to the fact that the parabola has finite dimensions with extreme symmetrical points K and K′ (Pic. 1).

We compose the normal equation to the point $K (X_K, Y_K)$ of the parabola and find the points of its intersection with the evolute, for which, solving together a system of these equations

$$\left.\begin{aligned} Y - Y_K &= -\frac{P}{X_K}(X - X_K) \\ Y &= P + \frac{3}{2}\cdot\sqrt[3]{P \cdot X^2} \end{aligned}\right\} \tag{4}$$

we get

$$-\frac{PX}{X_K} + Y_K = \frac{3}{2}\cdot\sqrt[3]{P \cdot X^2}.$$

Replacing, according to (1) Y_K, we write

$$\frac{X_K^2}{2P} - \frac{PX}{X_K} = \frac{3}{2}\sqrt[3]{PX^2}.$$

Raising the left and right sides of the last equation into a cube and performing the reduction of such terms, we obtain the cubic equation

$$\frac{P^3}{X_K^3}X^3 + \frac{15P}{8}X^2 + \frac{3X_K^3}{4P}X - \frac{X_K^6}{8P^3} = 0. \tag{5}$$

Denote the coefficients in front of the unknowns and the free term by *a*, *b*, *c*, and *d*, respectively. Having accepted

$$X = Z - \frac{b}{3a}, \tag{6}$$

and replacing X, we bring (5) to the “incomplete” form

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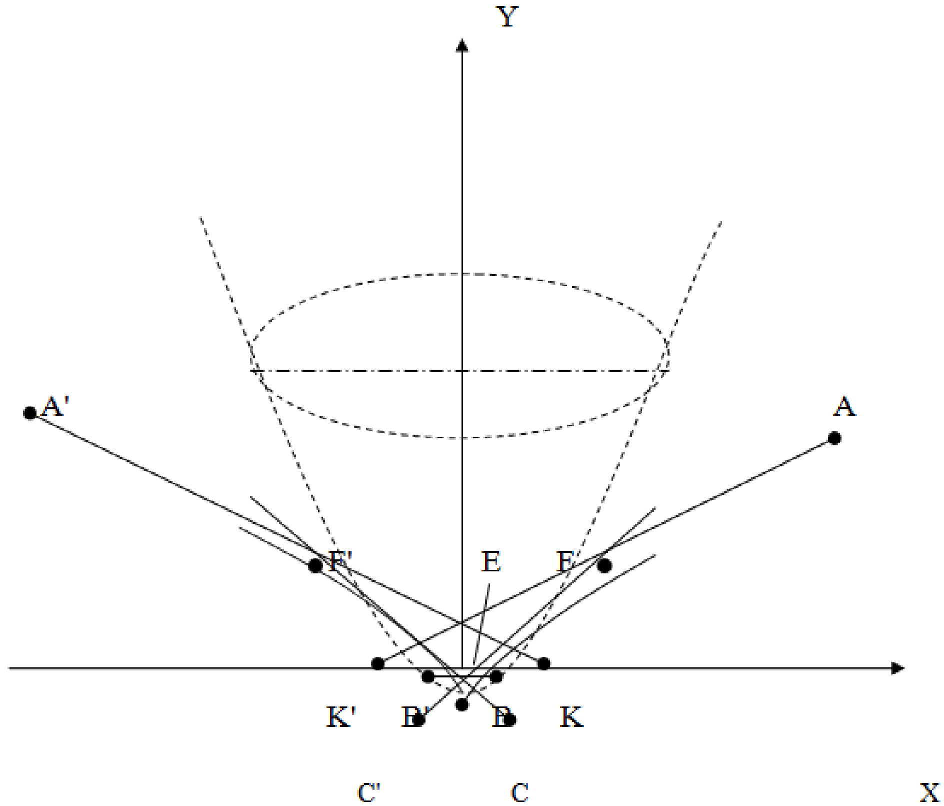
$$Z^3 + RZ + q = 0, \qquad (7)$$



Pic. 1: Layout of Normals to an Ideal Paraboloid Surface

According to (5), the roots of equation (7) are found by the formulas:

$$Z_1 = U + V; \qquad Z_{2,3} = -\frac{U+V}{2} \pm i\frac{U-V}{2}\sqrt{3}, \qquad (9)$$

in this

$$U = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{R^3}{27}}}; \quad \text{V}= \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{R^3}{27}}}.$$

To determine the value of the roots (9), we find the quantities included in U and V:

$$\frac{q^2}{4} = \frac{27^2 X_2^{18}}{4^9 P^{12}}; \qquad \frac{R^3}{27}=-\frac{27^2 X_2^{18}}{4^9 P^{12}}.$$

Therefore, the values of U and V are the same and equal:

$$U = V = \sqrt[3]{-\frac{q}{2} = \frac{3X_K^3}{8P^2}}. \qquad (10)$$

From here $Z_1 = \frac{3X_K^3}{4P}$, while X_1 according to (6) will be equal to

$$X_1 = \frac{X_K^3}{8P^2}. \qquad (11)$$

Substituting the obtained value of X_1 into equation (3) we find

$$Y_1 = P + \frac{3}{2}\sqrt[3]{\frac{X_K^6 P}{64P^4}} = P + \frac{3X_K^2}{8P}. \qquad (12)$$

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Using (9), we determine the remaining roots of equation (7). Since the values of U and V are the same, then

$$Z_{2,3} = -\frac{3X_K^3}{8P^2}.$$

We find from (6) the value $X_{2.3}$ of equation (5)

$$X_{2.3} = -\frac{3X_K^3}{8P^2} - \frac{15PX_K^3}{8P^3 \cdot 3} = \frac{X_K^3}{P^2}.$$

After substituting the found value in the equation of evolute (3) we get

$$Y_{2,3} = P + \frac{3}{2}\sqrt[3]{\left(-\frac{X_K^3}{P^2}\right)^2 P} = P + \frac{3X_K^2}{2P}.$$

Comparing the obtained values of $X_{2.3}$ and $Y_{2,3}$ with the values of x_c and y_c, we conclude that the mathematical calculations are performed correctly, and the values of $X_{2.3}$ and $Y_{2,3}$ determine the coordinates of the center, also points A of the evolute (3) of a circle in contact with a point *K* of a parabola.

Therefore, point B with coordinates X_1 and Y_1 is also located in evolution (3) and is the center of the contiguous circle for some parabola point (1) other than K.

Having compiled the equation of the tangent to the evolute (3) at point B (X_1, Y_1) and solving it together with the parabola equation (1), we determine the coordinates of the points of intersection of the tangent with the parabola.

$$\left.\begin{aligned} Y - Y_1 = \left(\frac{dY_Э}{dX}\right)(X - X_1) \\ Y = \frac{X^2}{2P}, \end{aligned}\right\} \qquad (13)$$

We find the value of the angular coefficient of the tangent to the evolution (3), for which we present its equation in the form:

$$Y_э = \frac{3}{2}P^{\frac{1}{3}}(X^2)^{\frac{1}{3}}.$$

From here

$$Y'_E = \frac{3}{2}P^{\frac{1}{3}} \cdot \frac{1}{3}(X^2)^{-\frac{2}{3}}2X = \frac{XP^{\frac{1}{3}}}{\sqrt[3]{X^4}} = \frac{\sqrt[3]{P}}{\sqrt[3]{X}}.$$

Since the angular coefficient is for point *B*, then substituting in the resulting expression instead of *X* its value from (11), we find

$$Y'_Э = \frac{\sqrt[3]{P}}{\sqrt[3]{\frac{X_K^3}{8P^2}}} = \frac{2P}{X_K}. \qquad (14)$$

Substituting the found value of the angular coefficient into the equation of the tangent to the evolution in (13) and replacing X_1, Y_1 and *Y* in it with their values from (11), (12) and (13), we obtain

$$\frac{1}{2P}X^2 - \frac{2P}{X_K}X - \left(\frac{X_K^2}{8P} + P\right) = 0. \qquad (15)$$

Define the discriminant of this quadratic equation

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$$D = \frac{4P^2}{X_K^2} + \frac{2}{P}\left(\frac{X_K^2}{8P} + P\right) = \frac{4P^2}{X_K^2} + \frac{X_K^2}{4P^2} + 2.$$

Since the value of the discriminant is D>0, the roots of equation (15) are real and different. Find the values of these roots

$$X_{1,2} = \frac{\frac{2P}{X_K^2} \pm \sqrt{\frac{4P^2}{X_K^2} + \frac{X_K^2}{4P^2} + 2}}{1/_P} = P\left(\frac{2P}{X_K} \pm \frac{4P^2 + X_K^2}{2PX_K}\right).$$

Substituting the found root values into the parabola equation, from (13) we obtain, respectively

$$Y_1 = \frac{4P^2}{Y_K} + 2P + \frac{Y_K}{4}; \qquad Y_2 = \frac{Y_K}{4}. \qquad (17)$$

Therefore, the tangent to the evolute at point B intersects the parabola at points F and C with coordinates X_1; Y_1 and X_2; Y_2 respectively.

IV. RESULTS

Considering in a similar way all mathematical calculations with respect to the point K' of a parabola with symmetric coordinates with respect to the point K, also with coordinates X'_K and Y'_K, we get the point A' corresponding to the coordinates of the center of curvature for the point K', the point B 'corresponding to the tangent to the evolution (3) at point A' with its other branch and point C' parabolas, defined by the intersection of the tangent to the evolute at point B' with a given parabola, also with a normal to the parabola at point C'.

It follows that when moving point B in a straight line to point B', we have a region BB' at the extreme points of which two normals to the parabola always intersect.

At any other point, for example E, of the segment BB', three normals to the parabola intersect. One of them coincides with the tangent drawn through point E to the AB' section of the evolute, the second to the section BA', and the third to the inner branch BVB' of the evolute closest to the point E (for point E, the section of the inner branch of the evolute is BB). If the point E is at the intersection of the straight line BB' with the y-axis of the parabola, then the third normal coincides with the axis of the parabola. When the point E is closer to the point B', three normals to the sections BA', A'B 'and B'V evolutes are also formed.

V. CONCLUSION

As a result of the analysis, the following conclusions can be drawn:

1. For any parabola symmetrically bounded by a secant line perpendicular to its axis, there exists a region with minimal dimensions in which all normals intersect.
2. The boundaries of this region are determined by the intersection of the normals to the four points of the parabola, two of which are the extreme points of the parabola K (X_K, Y_K), K'($-X'_K$, Y'_K), and the coordinates of the other two are C and C' are calculated as:

$$X_{C,C'} = \pm\frac{X_K}{2}; \qquad Y_{C,C'} = \frac{Y_K}{4}. \qquad (18)$$

3. At any point in this region, three normals to the parabola are suppressed.

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4. The distance D from the top of the parabola to the center of this region is calculated by the formula:

$$D = P + \frac{3X_K^2}{8P}. \quad (19)$$

5. The length *l* of this area is calculated by the formula

$$l = \frac{X_K^3}{4P^2}. \quad (20)$$

6. The coefficient Q of the decrease of the aperture (distance KK′) of the parabola during transition to the length *l* of the minimum area is calculated by the formula

$$Q = \frac{8P^2}{X_K^2} = \frac{4P}{Y_K}. \quad (21)$$

The obtained parabola properties can be used to optimize the alignment process of the concentrator.

REFERENCES

1. Zakhidov R.A. Mirror systems of concentration of radiant energy. Tashkent, Fan, 1986.
2. Zakhidov R.A., Umarov G.Ya., Weiner A.A. Theory and calculation of solar systems. Fan. Tashkent, 1977.
3. Avchiev Sh.K. Methods and means of geodetic support for the adjustment of solar energy concentrates. Fan wa technology. Tashkent, 2019.
4. Avchiev Sh.K. Alignment and control of the facets of concentrators of a solar high-temperature installation // Architecture. Construction. Design. - Tashkent. 2016, No. 1, p. 53-56.
5. Trombe F., Gion L, Royere C, Robert I.F. First results obtained with the 1000 kw solar Furnace. Solar Energy, 1975, 15.
6. Trombe F., Le Phat Vih A. Thousand kw solar furnace bult by the National Central of Scientific Research in Odeillo (France). J. Solar Energy", 1975,VO1.15, 1,P.57-65.
7. Royere C. Le applications du for solaire de 1000 kw du CNRS a Odiello. “Entropie”, 1981, vol.17, 97, P.147-160.
8. Avchiev SH.,Islamova Z., Avchiev M. ”Pre-Calculation of The Accuracy of Geodetic Works During Installation And Adjustment of Solar Furnaces” International Journal of Computer Science and Engineering. ISSN 2348-8387, VOL. 6, Issue 5, May 2019. (Global Impact Factor,IF: 2.15).
9. James M. Les Fours Solaires du Laboratories central de l*Armement. Rev. int. Heliostat. Temp, et Refract., 1975, vol.
10. Bronstein I.N., Semendyaev K.A. Math reference. M., Science, 1980.
11. Filchakov P.F. Handbook of Higher Mathematics. - Kiev: Nukova Dumka, 1973.
12. Tsvetkov K.A. Practical astronomy. M., Geodesizdat, 1981, 528 p.

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