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Stally Estimation of the Initial Trajectory of Motion in the Problem of Identification of Dynamic Systems Based on the Quasilinearization Method

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***Abstract*---*The problems of formation and construction of stable algorithms for estimating the initial trajectory of motion in the problem of identification of dynamic systems based on the quasilinearization method are considered. In constructing stable computational estimation procedures, a projection algorithm is used to solve systems of linear algebraic equations based on the conceptions of the matrix pseudo-inversion. The given stable algorithms are very effective in solving practical problems of identifying control objects in real time.*

***Key words*---*dynamic system, identification, quasilinearization, initial trajectory of motion, stable estimation.*

I. INTRODUCTION

Nowadays, there are various methods of identification, based on different approaches to the form of identification models. To build control systems for dynamic objects for which identification is part of the adaptation process, the choice of a mathematical method of identification is an important issue. To date, a large number of papers have been published in which various approaches, methods, algorithms, and computational schemes are used to identify objects [1–7]. To apply these methods, it is necessary to have as an a priori information an equation of the object model, in which only parameters can be unknown.

One of the most effective methods for estimating and identifying dynamic systems is the indirect computational method, known as the quasilinearization method [8]. The quasilinearization method is essentially a method of transforming nonlinear multipoint boundary value problem, which is basically stationary, into a linear non-stationary problem. The nonstationarity of parameters can also be taken into account, if they change slowly in comparison with the rate of convergence of the identification procedure. The method is essentially iterative; it does not require the introduction of special test actions and is therefore applicable for use in real time.

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II. Formulation of the problem

We consider the quasilinearization algorithm for a discrete system. Let a nonlinear nonautonomous *N*-dimensional difference equation be given

$$\gamma(k+1) = \varphi[\gamma(k), k] \tag{1}$$

restrictions are given in the form:

$$C(k_j)\gamma\ k_j = b_j, (j = 1,2,...m)\,. \tag{2}$$

It is necessary to find the trajectory $\gamma(k), k \in [k_0, k_f]$, which satisfies equation (1). We assume that $k_j < k_k$ if $j < i$ and conditions (2) are consistent with a unique solution of equation (1). The solution of the initial boundary value problem is approximated by the initial trajectory $\gamma^i(k)$. For select $\gamma^i(k)$ use the linear terms of the expansion in a Taylor series:

$$\gamma^{i+1}(k+1) = \varphi[\gamma^i(k), k] + \frac{\partial\varphi[\gamma^i(k), k]}{\partial\gamma^i(k)}[\gamma^{i+1}(k) - \gamma^i(k)]\,.$$

After transforming this expression, we obtain a linear nonautonomous inhomogeneous difference equation for the new approximation $\gamma^{i+1}(k)$:

$$\gamma^{i+1}(k+1) = \frac{\partial\varphi[\gamma^i(k), k]}{\partial\gamma^i(k)}\gamma^{i+1}(k) + \left[\varphi[\gamma^i(k), k] - \frac{\partial\varphi[\gamma^i(k), k]}{\partial\gamma^i(k)}\gamma^i(k)\right]$$

the solution of this equation is

$$\gamma^{i+1}(k) = \Omega^{i+1}\gamma^{i+1}(k_0) + p^{i+1}(k)\,, \tag{3}$$

where $\Omega^{i+1}(k)$ is the solution to the equation

$$\Omega^{i+1}(k+1) = \frac{\partial\varphi[\gamma^i(k), k]}{\partial\gamma^i(k)}\Omega^{i+1}(k)\,.$$

With the solution initial condition $\Omega^{i+1}(k_0) = I$, a $p^{i+1}(k)$ – is the solution of the equation

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$$\dot{p}^{i+1}(k+1) = \varphi[\gamma^i(k), k] - \frac{\partial\varphi[\gamma^i(k), k]}{\partial\gamma^i(k)}[\gamma^i(k) - p^{i+1}(k)]$$

with the initial condition $p^{i+1}(k_0) = 0$.

It follows that the problem of building $\gamma^{i+1}(k)$ reduces to solving two simple difference equations with given initial conditions, and the problem parameter $\gamma^{i+1}(k_0)$ must be chosen so that conditions (2) are satisfied. To do this, we substitute the expression (3) in to the conditions (2):

$$C(k_j)[\Omega^{i+1}\gamma^{i+1}(k_0) + p^{i+1}(k)] = b_j \quad (j = 1,2,...,m)$$

and rewrite in the form of a system of m linear algebraic equations

$$[C(k_j)\Omega^{i+1}(k_j)]\gamma^{i+1}(k_0) = b(k_j) - C(k_j)p^{i+1}(k_i),$$

$$(j = 1,2,...,m)$$

or in the vector form

$$A\gamma^{i+1}(k_0) = b \tag{4}$$

where

$$b = \begin{bmatrix} b(k_j) - C(k_1)p^{i+1}(k_1) \\ ..... \\ b(k_2) - C(k_2)p^{i+1}(k_2) \\ ..... \\ ..... \\ b(k_m) - C(k_m)p^{i+1}(k_m) \end{bmatrix} \quad A = \begin{bmatrix} C(k_1)\Omega^{i+1}(k_1) \\ ..... \\ C(k_2)\Omega^{i+1}(k_2) \\ ..... \\ ..... \\ C(k_m)\Omega^{i+1}(k_m) \end{bmatrix}.$$

After solving the equation (4), we can find the initial condition $\gamma^{i+1}(k_0)$, in which we can determine the new approximation to the true trajectory of motion.

One of the computational difficulties associated with solving equation (4) is the degeneracy or poor conditionality of the matrix M^{i+1}. This most often happens on the first steps of the algorithm, when the next approximation of the trajectory is still not a good estimate of the solution.

To solve the system of equations (4), you can use any of the known methods for solving systems of linear equations. It can be shown [9-13] that when solving such equations, the methods based on the well-known

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Kachmazh iterative algorithm using pseudoinverse matrices turn out to be very effective. The use of these matrices makes the algorithm practically convenient and suitable for use in a wide range of ratios between the required and available volumes of computer RAM, makes it easier to control the accuracy of intermediate calculations and, if necessary, regularize the calculated solution.

III. SOLUTION

With sufficiently small errors in the source data, the first approximations in the iterative methods usually differ little from the corresponding approximations with the exact source data, and therefore it initially converses to the desired solution. With the increase in the number of iterations, the obtained approximations can arbitrarily far deviate from the desired solutions. Therefore, iterative regularization [14-16] consists in that, according to the available a priori information, in particular on the error of the initial data, we choose from the entire sequence some approximation close enough to the original solution. In other words, for the principle applicability of any iterative method, to the solution of ill-posed problems, it must generate a regularizing family of operators, in which the regularization parameter is the iteration number. A necessary condition for this is the convergence of the method with exact initial data.

In accordance with the general definition of a regularizing operator according to A.N. Tikhonov, we say [15] that the iterative method

$$k_{pq}^{r+1} = F\left(k_{pq}^{r}, \lambda\right), \qquad r = 0,1,2,..., \tag{5}$$

generates a regularizing family of operators, in which the parameter is the iteration number, if for any initial approximation k_{pq}^{0} and for any σ, satisfying the condition $0 < |\sigma| < |\sigma_0|$, there is a number $N = N(\sigma)$, such that $\lim\limits_{\sigma \to 0} k_{pq}^{N(\sigma)} = k_{pq} \in K_{\lambda}$, where λ – is the vector of the input data of the problem. That is, if there is such a method for choosing an approximation from the iteration sequence (5), in which the obtained approximations converge to the exact solution when the error of the initial data σ tends to zero.

In [9-13], it is emphasized that the Kachmazh method is equivalent to the Gauss – Seidel algorithm with respect to some new unknown vector obtained after applying the right Gauss transformation to the original system.

In [17], it is noted that the generalized Kachmazh methods can be competitive due to their exceptional universality in the class of systems with “very bad” matrices, where many other algorithms are inapplicable. In addition, the development and study of block versions of the Kachmazh method, in which the inclusion of special orderings of unknowns can significantly reduce the total number of arithmetic operations, is perspective.

A number of authors [9,13,17,18] investigated additive projection methods for solving SLAEs based on geometric approaches close to the Kacmage method, which is a multiplicative projection algorithm according to the classification from [19]. As shown there, additive iterative processes have, generally speaking, a worse convergence

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rate. However, it is obvious that the simultaneous finding of projections on different hyperplanes, on which projection additive methods are based, makes it especially easy to parallelize computations when implemented on multiprocessor computer systems.

Using pseudo-inverse matrices, one can lead to an algorithm for solving a system of linear equations based on the use of the following recurrent formula [13,17]:

$$\gamma_k^{i+1}(k_0) = \gamma_{k-1}^{i+1}(k_0) + A_k + [b_k - A_k\gamma_{k-1}^{i+1}(k_0)]$$
$$\gamma_0^{i+1}(k_0) = \gamma_0(k_0)\ \ k = 1,2,...,m \qquad (6)$$

where $\gamma^0(k_0), \gamma_{k-1}^{i+1}(k_0), \gamma_k^{i+1}(k_0)$ are initial arbitrarily given approximate solution and solutions computed on $(k\text{-}1)$ -th and *k*- th steps respectively; A_K, b_k are an $m \times n$ matrix and an *m*-dimensional vector formed from rows of matrix *A* and *a* component of vector y in accordance with the formulas

$$a_i^k = a_p, b_k = b_p, i = 1,2,...,m$$

where $p = i + km - n[(i + km - 1)/n]$; *m*-is some natural number, $[a]$ – the integer part of number; $a_i^k, b_i^k, a_p, \gamma_p$ the essence of the *i*-th and *p*-th rows (components) of matrices (vectors) A_k and A (b_k and b) respectively; A_k^+ –is a pseudo inverse to the matrix A_k, which can be calculated by recursive formula:

$$A_{k1}{}^+ = (a_1^k)^T (a_1^k, a_1^k)^{-1}$$
$$A_{k,j}^+ = (A_{k,j-1}^+ | 0) - D_j \square\square\square j = 2,.3,...,m$$
$$D = \begin{cases} (a_i^k - z_j A_{k,j-1})^+ d_j & a_i^k \neq z_j A_{k,j-1} \\ [1 + (z_j, z_j)]^{-1} A_{k,j-1}^+ z_j d_j & a_i^k = z_j A_{k,j-1} \end{cases},$$

where A_{kJ} -is a $j \times n$ -matrix composed of the first *j* rows of the matrix $A_{k1} = a_1^k$, $A_{km} = A_k$, $A_{kj}{}^+$-is an $n \times j$ matrix, pseudoinverse to $A_{kj} - z_j = a_i^k A_{k,j-1}^+ d_j = (z_j | -1)$, $(\cdot|\cdot)$ -is a block form of matrix, the vertical bar in which separates the last column of the matrix; $(\cdot|\cdot)$, *m* – are the symbols, respectively, of the scalar product of vectors and the transposition of vectors and matrices.

The process of calculating the solutions x_k by the formula (6) continues until the solutions calculated on two adjacent steps be equal to each other, i.e. until the condition is satisfied, for example

$$\left\| \gamma_k^{i+1}(k_0) - \gamma_{k-1}^{i+1}(k_0) \right\| \leq \varepsilon,$$

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where $\left\|\gamma_k^{i+1}(k_0)-\gamma_{k-1}^{i+1}(k_0)\right\|$ -is any of the norms of the vector $\left\|\gamma_k^{i+1}(k_0)-\gamma_{k-1}^{i+1}(k_0)\right\|$, ε - is some sufficiently small positive number chosen taking into account the necessary accuracy of the desired solution. The foregoing fully describes the sequence of calculations of the solution x of system (4) in accordance with the algorithm under consideration.

IV. CONCLUSION

The above regular algorithms are very effective in solving practical problems of generalized estimation and identification of control objects in real time.

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