

GRAPH LABELING AS A USEFUL MODEL FOR A WIDE VARIETY OF APPLICATIONS

Madhuchanda Rakshit¹, Daljeet Kaur²

^{1,2}Guru Kashi University, Talwandi Sabo

ABSTRACT

We may all be familiar with graphs and their relevance. In recent years, graph labelling has become equally important. The current research focuses on several graph labelling applications. Alexander Rosa was the first to present graph labelling in the 1960s. Currently, a variety of graph labelling strategies exist, and a vast amount of literature on various graph labelling challenges is available in both printed and electronic form. Graph theory is linked to a variety of fields of mathematics and sociology, including discrete mathematics, network theory, information framework, and interpersonal organisation research. In graph theory, there are various exciting areas of study. The labelling of graphs is one of these areas that cuts across a wide range of applications.

KEYWORDS: *Graph Labeling, Problems, Graph theory, applications, mathematics*

1. INTRODUCTION

A graph labelling task involves assigning numbers to vertices, edges, or both, under certain conditions. In the mid-1960s, graph labelling was first introduced. Over 200 graph labelling strategies have been studied in during 2500 papers over the last 50 years. Due to the enormous amount of documents and the fact that a substantial number of the papers have appeared in diaries that are not widely available, determining what has improved the situation and staying informed of fresh revelations is difficult. In this overview, I've compiled what I've learned about graph labelling as a useful paradigm for a variety of applications.

Currently, a limited number of graph labelling approaches are available. In the late 1960s, a problem in radio space science prompted the task of examining unmistakable districts of the divine vault using total contrasts of pairs of numbers occurring on the places of radio receiving wires to the connections of the format designs of the reception apparatuses under the limitations of the ideal design to formulate more brief

mathematical problems on graph labelling. After a setting of scoring a bar in a workshop piqued the interest of many eager minds, S. W. Golomb used the term "agile numbering" to describe an independent discovery of value in 1972. The difficulties highlighted by graph labelling pose a challenge to our brain's ability to solve them.

2. GRAPH LABELING

A diagram $G = (V,E)$ is comprised of two limited sets: $V(G)$, which is a nonempty assortment of components called vertices, and $E(G)$, which is a set (possibly vacant) of components called edges. A chart can be considered at the time as a drawing or outline comprised of an assortment of vertices (spots or focuses) and edges (lines) associating explicit pairings of these vertices. Figure 1 shows a chart $G = (V,E)$ with $V(G) = \{ v_1, v_2, v_3, v_4, v_5 \}$ and $E(G) = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7 \}$.

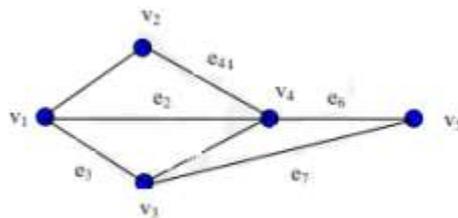


Figure 1: A graph G with five vertices and seven edges

The two vertices with which an edge interacts are referred to as the edge's vertices. Figure 1 shows $e_1 = (v_1, v_2)$ and $e_2 = (v_1, v_4)$. An edge e of chart G is supposed to be related with the vertex v assuming v is the end vertex of e . For instance, in Figure 1, an edge e_1 with two vertices v_1 and v_2 is shown. An edge with vague end vertices is known as a circle. Toward the day's end, an edge e is a circle that interfaces a vertex v to itself. The level of a vertex v , which is developed of d , is the quantity of edges that happen with v . (v). In Figure 1.1, we have $d(v_1) = 3$, $d(v_2) = 2$, $d(v_3) = 3$, $d(v_4) = 4$, and $d(v_5) = 2$. G is supposed to be k -standard if $d(v) = k$ for every vertex v of chart G for some sure entire number k . A chart G is supposed to be connected in the event that there is a way interfacing each sets of vertices. Whenever there is no stress over the course of an edge, the chart is called undirected. An undirected and related chart is displayed in Figure 1. Chart hypothesis, dissimilar to most different parts of math, has a strong groundwork, tracing all the way back to 1707, when Swiss mathematician Leonard Euler (1707-1783) investigated the seven Konigsberg ranges. During the eighteenth century,

the Pregel waterway split the city of Königsberg (in Prussia) into four portions. Seven scaffolds connected these districts, as seen in Figure 2. (a). A, B, C, and D each have their own section. According to legend, the Königsberg residents had a good time trying to discover a path that ran through each extension only once (It was OK to go to a similar island any number of times).

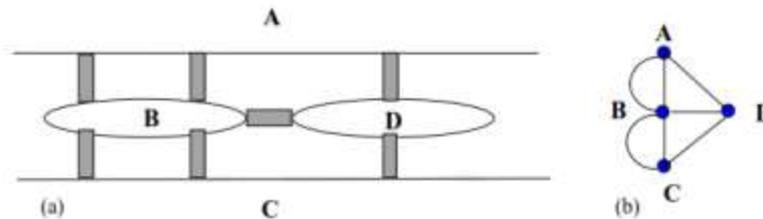


Figure 2: (a) A map of Königsberg (b) A graph representing the bridges of Königsberg

3. ROLE OF GRAPH LABELING

Several applications are listed in this section. A different type of graph is utilised to depict the problem for each type of application, based on the problem scenario. To fix the challenge, a suitable labelling is applied to that graph. Various topics are considered, beginning with the establishment of quick, effective communication.

3.1 Fast Communication in sensor networks Using Radio Labeling

Each station is given a channel (a positive integer) to avoid interference when a group of transmitters is used. The stronger the interference gets as the distance between stations decreases, As a result, the channel assignment difference must be higher. In this picture, every vertex addresses a transmitter, and each sets of vertices associated by an edge indicates neighboring transmitters. Let $G = (V(G), E(G))$ be an associated diagram, and the distance between any two vertices in G be $d(u, v)$. The greatest distance between any two vertices is given by G 's measurement, $\text{diam}(G)$. For any vertices u and v , a radio naming (or multi-facet distance marking) for G is a \mathbb{N} association f with the end goal that $|f(u) - f(v)| \geq d(u, v) + 1$.

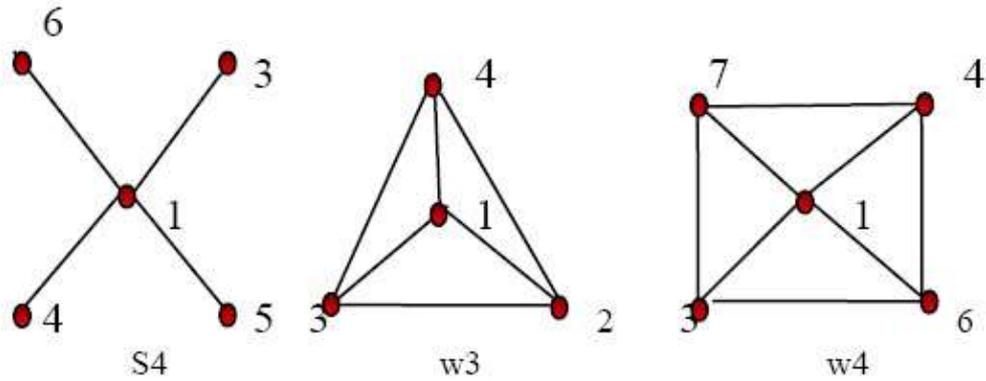


Fig 1. Radio Labeling on different kind of Graphs

Fig1: Radio Labeling on different kind of Graphs

For any application, the radio labelling procedure has proven to be a reliable method of measuring the communication time for sensor networks.

In this case, the network is modelled as a chain graph, with each sensor acting as a At time t , the vertex communicates, where t is the radio channel assignment. The random dump of garbage in the network, followed by radio labelling, was discovered to have the property of having "consecutive" channel assignments – close temporal frames. separated by a significant distance. The time at which sensors communicate can be determined using channel labelling.

3.2 Designing Fault Tolerant Systems with Facility Graphs

An office diagram is utilized to portray the organization in this application. An office diagram is a G with hubs addressing framework offices and edges addressing office access connections. An office is an equipment or programming part of any framework that can come up short all alone in this specific circumstance. Equipment offices incorporate control units, number-crunching processors, stockpiling units, and info/yield hardware. Programming offices incorporate compilers, application programs, library schedules, and working frameworks. Since every office can get to different offices, the constant frameworks are addressed as an office chart.

The act of assigning office types with numbers in enclosure is known as chart vertex marking. The diagram shows the many kinds of offices that are utilized by different offices. The x_1 hub can speak with the x_2 , x_3 , and x_4 hubs. Also, hub x_5 with office type

t1 approaches office types t3, t1, and t2 on hubs x3, x2, and x4. There is compelling reason should be worried about the correspondence interface assuming a hub in this office chart falls flat since the office diagram will observe one more channel and the correspondence interaction will go on as in the past.

3.3 Automatic channel allocation for small wireless local area network.

In industries such as cellular phones, Wi-Fi, security systems, and many more, secure transmissions are necessary others in order to discover an efficient solution. It's inconvenient to be on the phone and have another person on the line with you. Interferences induced by unrestricted simultaneous broadcasts cause this annoyance. When two channels are near enough together, they might interfere or resonance, causing communications to be harmed. Interference can be prevented by adopting the proper channel assignment.

The task of allocating a nonnegative integer channel to each TV or radio transmitter deployed in separate locations so that communication is not disturbed is known as the channel assignment problem. In a diagram model of this issue, the transmitters are addressed by the vertices of a chart; two vertices are incredibly close assuming that they are adjacent in the diagram, and close assuming they are isolated by two.

Close emitters must receive distinct channels, while extremely close broadcasters must receive channels that are at least two apart, in the channel assignment conundrum private communication.

This challenge is solved by modelling the wireless LAN network as an interface graph and utilising the graph labelling technique to solve it.

The access points (vertices) in an interference graph interfere with other access points in the same region. The graph is known as an interference graph, and it is made up of nodes that are access points. If the nodes interact with one other when using the same channel, an undirected edge connects them. The channel allocation problem has now been transformed into a graph labelling problem, often known as a vertex labelling problem.

A vertex colouring C is the collection of colours that correspond to the access points' channels, and $f: v(G) \rightarrow C$ is the function. There should be no overlapping edges in these

channels. The labelling algorithm DSATUR (Degree of Saturation) is utilised for labelling. It's a heuristic search, which means it's looking for vertices with the most varied coloured neighbours. Assuming this subset contains just a single vertex, that vertex is picked for naming. Assuming the subset contains more than one vertex, the marking is done in diminishing request of unlabeled neighbors. Assuming more than one applicant vertex is accessible, a deterministic determination work is utilized to pursue a definitive choice. To recognize the neighbors, the convention activity is done by paying attention to the messages created by the passageways. The convention activity is finished when the passageways rebroadcast a message. The impedance chart is then created, and the it is applied to name process. The channels and the chart have a comparable relationship in that the channels pay attention to messages at normal spans, and the graph does the same. labelling algorithm should be run at regular intervals as well.

3.4 Avoiding Stealth Worms by Using Vertex Covering Algorithm

Given a basic diagram G with n vertices named $1, 2, \dots, n$, the vertex cover calculation searches for a vertex front of size all things considered k . Assuming the made vertex cover has a size of at minimum k , stop at each stage.) is an instrument that recreates the spread of covertness worms on huge PC networks continuously and produces the best infection security strategies. It's basic to track down worm engendering to prevent them from spreading continuously.

The fundamental thought is to make an organization with a base vertex cover, with vertices addressing directing servers and edges addressing associations between them. Then there's the best worm propagation solution is discovered.

3.5 Analyzing Communication Efficiency in sensor networks with Voronoi Graph

Sensor networks can be used for a wide range of purposes. Mobile object tracking, environmental data collection, defence applications, health care, and so on...

To analyse the communication efficiency, the sensor network is modelled as a graph. The sensor network is modelled using a voronoi graph. Because the voronoi graph is built in the shape of polygons in a plane, the nodes can be thought of as sensors, and the polygon boundaries can be thought of as the sensing range of each sensor.

The detecting range of these sensors can be thought of as a polygon. One of these sensors will serve as the cluster head for reporting purposes. If the sensing ranges of two sensors share a same border in the voronoi graph, they are considered neighbours.

When things move across the detecting scope of one sensor and into the detecting scope of another sensor, the previous sensor ought to hand-off this to the abutting sensor properly.

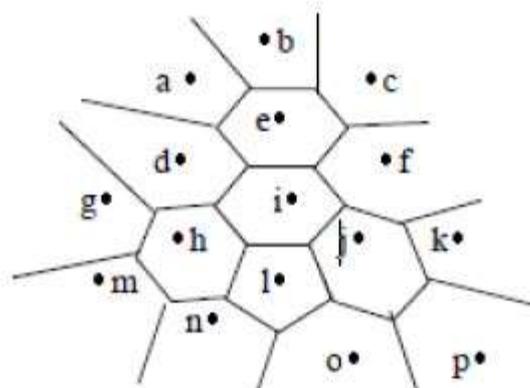


FIG 3. Vornoi Graph

CONCLUSION

This study's main purpose is to look into the role of graph labelling in the field of communication. Graph Labeling, as previously stated, is a powerful approach that makes things easier in a variety of networking domains. The goal of this overview is to convey the Graph Labeling concept. Researchers can learn about graph labelling and its uses in the communication industry, as well as get some inspiration for their own research. The vertex v of a graph G is known as a cut-vertex of G if the evacuation of v expands the number of components. The edge e of a graph G is known as an if the evacuation of e increases the number of components. cut edge or extension. If the number of segments of $G - S$ is greater than that of G , the number of edges S is called an edge cut of G . A graph's square is a non-trivial, maximally related subgraph with no cut-vertices.

REFERENCES

1. L. Pandiselvi, S. NavaneethaKrishnan and A. NellaiMurugan, Path Related V4 Cordial Graphs. International Journal Of Recent Advances in Multidisciplinary Research, Vol. 03, Issue 02, pp.1285-1294, February 2016.

2. A. Rosa, On certain valuations of the vertices of a graph, Theory of graphs (International Symposium, Rome), July (1966).
3. C. Sekar, Studies in Graph Theory, Ph.D.Thesis, Madurai Kamaraj University, 2002. R. Sridevi, S. Navaneethakrishnan and K. Nagarajan, Odd-Even graceful graphs, J.Appl.Math.& Informatics Vol.30 (2012), No. 5-6, pp. 913-923.
4. M. Sundaram, R. Ponraj and S. Somasundaram Prime Cordial Labeling of graphs, Journal of Indian Academy of Mathematics, 27 (2005) 373-390.
5. R. Sridevi, A. NellaiMurugan, A. Nagarajan, and S. Navaneethakrishnan Fibonacci Divisor Cordial Graphs, International Journal of Mathematics and Soft Computing.
6. R. Sridevi, S. Navaneethakrishnan Some New Graph Labeling Problems and Related Topics (Thesis). Manonmaniam Sundaranar University, January 2013. Solomon W. Golombo, How to number a graph, Graph Theory and Computing, Academic Press, New York (1972), 23-37.
7. R. Tao, On k-cordiality of cycles, crowns and wheels, Systems Sci., 11 (1998), 227-229. R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Special classes of Divisor Cordial graphs, International Mathematical Forum, vol.7, 2012, no.35, 1737-1749.
8. R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Divisor cordial graph, International Journal of Mathematical Comb., Vol .4(2011),15-25.
9. S.K. Vaidya, U.M. Prajapati Fibonacci and Super Fibonacci Graceful Labelings of some cycle related graphs, International J.Math.Combi. Vol.4, (2011), 59-69.
10. R. Vasuki, A.Nagarajan Studies in Graph Theory – Labeling Problems in Graphs (Thesis), Manonmaniam Sundaranar University, December 2010. M.Z. Youssef, On k-cordial labeling, Australas .J. Combin., Vol 43(2009), 31- 37.
11. G.J. Gallian, A Dynamic survey of graph labeling, The electronic journal of combinatorics, 16 (2009), #DS6. Z. Gao, Odd graceful labelings of some union of graphs, J. Nat. Sci. Heilongjiang Univ, 24(2007) 35-39.
12. R.B. Gnanajothi, Topics in Graph Theory, Ph.D. Thesis, Madurai Kamaraj University, 1991. S.W. Golombo, How to number a graph in graph Theory and Computing, R.C.Read, ed., Academic Press, New York (1972) 23-37.