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Construction of Neighbour Balanced Design with Parameters $v = 2t + 1 = b$, $r = t = k$, $\lambda = \frac{t-1}{2}$, where t is an Odd Number

Sarita Saini[1]
saritasaini15@gmail.com

R. R. Laxmi[2]
Professor, Department of Statistics,

M. D. University, Rohtak, Haryana, India
drrrlaxmi@gmail.com

Abstract: Let x be the primitive element of GF $(2t + 1)$ where v (number of treatments) is a prime number. Let $v = 2t + 1$, where t is an odd positive integer number. In this paper, a Neighbour Design from the BIBD using the method of Galois field with the parameter $v = 2t + 1 = b, r = t = k, \lambda = t - 1/2$ is constructed with the initial block $(x^0, x^2, x^4, \dots \dots, x^{2t-2})$ and the remaining $(v - 1)$ blocks are obtained by developing initial block cyclically with mod (v).

Keywords: Galois Field, Border Plots, Neighbour Design, Left & Right Neighbours, Neighbour Balanced Design.

1. INTRODUCTION

According to Rees (1967), neighbour design is a collection of circular blocks in which each pair of distinct treatments appears as nearest neighbour equally often. Azais et al. (1993) in agriculture, horticulture and forestry experiments oftenly showed that the response on a given plot is affected by the treatments on neighbouring plots as well as by the treatment applied to that plot. Laxmi and Rani (2009) constructed neighbour designs using left and right border plots from OS_1 series and obtained two sided neighbour treatments for every treatment of the Design. Laxmi and Parmita (2010, 2011) constructed Neighbour Designs from OS_2 series and suggested a method of finding left and right neighbours of a treatment without observing the original design. Singh, K. K. and Meiti (2011) defined the one sided circular neighbour balanced designs. Laxmi and Kumar (2017) constructed the partially neighbour balanced complete block designs for $v = s - 1$ using co-primes to prime numbers s. Kumar and Laxmi (2019) further constructed the partially neighbour balanced complete block designs through Latin Square with parameters $v = s - 1 = b$, $r = s - 1 = k$, $\lambda_r = \lambda_c = 2$ and $\lambda'_r = \lambda'_c = 4$.

2. Construction of Neighbour Balanced Incomplete Block Design:

Let $v = 2t + 1$be a prime number, where $t$is any odd positive integer number. Let$x$ be the primitive element of GF ($v = 2t + 1$). The block suggested as the Initial Block consisting of following elements with $mod(v)$ for the construction of Block Design.

$$\boldsymbol{I = \{x^0, x^2, x^4, \dots \dots \dots \dots \dots \dots \dots \dots \dots, x^{2t-2}\}}$$

The initial block contains t elements, so the size (k) of the Initial Block of the design is t which is less than v, the number of treatment and hence the design shall be as Incomplete Block Design . This initial block when developed under $mod(v)$ generates a series of Block of the design using the cyclic shift method.. As the design obtained using cyclic shift method therefore, each treatment is replicated as may times as the size of the block i.e. $r = t = k$ and as many locks are constructed as the number of treatment are i.e. $v = b$. Since the block size is t, so there can be $t_{c_2} = t(t-1)/2$ number of pairs of treatment block. As each treatment in not occurring in all the blocks rather appearing in t blocks. So it will for {t(t-1)/2}/t number of pairs of treatments in the complete design i.e. $\lambda = (t - 1)/2$, since $v = b = 2t + 1, r = k = t$ and $\lambda = t - 1/2$. So the design constructed using the suggested Initial Block is a Balanced Incomplete Block Design from which Neighbour Design can further can be obtained by using border plots method for different values of t. In this paper, construction of Neighbour Design with different values of v, where t is only odd positive integer number is discussed and the neighbours of a treatment are also found for these designs, which are given in tabular form.

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For $t = 5$

Let us consider $\boldsymbol{t = 5,}$ so that $\boldsymbol{v = 2t + 1 = 11}$ (prime number). The primitive roots of the GF(11) are $\{2, 6, 7, 8\}$. The initial block developed with primitive root $x = 2$ is $\{2^0, 2^2, 2^4, 2^6, 2^8\}$ with mod (11) or $\{1, 4, 5, 9, 3\}$.The Initial Block with primitive root $x = 6$ is $\{6^0, 6^2, 6^4, 6^6, 6^8\}$ with mod (11) or $\{1, 3, 9, 5, 4\}$. The Initial Block with primitive root of $x = 7$ is $\{7^0, 7^2, 7^4, 7^6, 7^8\}$ with mod(11) or $\{1, 5, 3, 4, 9\}$. The Initial Block with primitive root of $x = 8$ is $\{8^0, 8^2, 8^4, 8^6, 8^8\}$ with mod(11) or (1, 9, 4, 3, 5).

It is to be noted here that in all these Initial Blocks, the treatments are same so the design (D) of interest with $v = 11$ can be constructed using any of these initial blocks. Here design is constructed using the initial block (1, 4, 5, 9, 3) obtained from primitive root $x = 2$. The remaining blocks of the design are obtained by using the cyclic shift method under $mod(11)$ through initial block.

This design is the symmetric BIB Design with the parameters$v = 11 = b, r = 5 = k, \lambda = 2$, which can be written as $v = 2t + 1 = b, k = t$with replication of each treatment being t& $\lambda = t$-1/2. From this design D the neighbour design is obtained using the method of border plots. The neighbour design D^* thus obtained from D can be written as

$D^* =$

3	1	4	5	9	3	**1**
4	2	5	6	10	4	**2**
5	3	6	7	11	5	**3**
6	4	7	8	1	6	**4**
7	5	8	9	2	7	**5**
8	6	9	10	3	8	**6**
9	7	10	11	4	9	**7**
10	8	11	1	5	10	**8**
11	9	1	2	6	11	**9**
1	10	2	3	7	1	**10**
2	11	3	4	8	2	**11**

Left and Right Neighbours of a Treatment of the Design

The left & right neighbours of treatments for the neighbour design, when $\boldsymbol{t = 5}$, with parameters $v = 11 = b,\ r = 5 = k$ are summarized in the following table:

Table 1

Other Left Neighbours	Left Neighbours	Treatment Numbers	Right Neighbours	Other right Neighbours
3, 11	7, 8, 9	1	4, 5, 6	2, 10
4, 1	8, 9, 10	2	5, 6, 7	3, 11
5, 2	9, 10, 11	3	6, 7, 8	4, 1
6, 3	10, 11, 1	4	7, 8, 9	5, 2

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7, 4	11, 1, 2	5	8, 9, 10	6, 3
8, 5	1, 2, 3	6	9, 10, 11	7, 4
9, 6	2, 3, 4	7	10, 11, 1	8, 5
10, 7	3, 4, 5	8	11, 1, 2	9, 6
11, 8	4, 5, 6	9	1, 2, 3	10, 7
1, 9	5, 6, 7	10	2, 3, 4	11, 8
2, 10	6, 7, 8	11	3, 4, 5	1, 9

Here it is observed that each treatment has once all other treatments as either a left neighbour or a right neighbour. So it is a design which is balanced for neighbours with $\boldsymbol{\lambda}' = \mathbf{1}$, where $\boldsymbol{\lambda}'$is the treatment pairs occur together as either a left neighbour or a right neighbour.

It can be observed for the other value of $\boldsymbol{t = 11,}$ so that $\boldsymbol{v = 2t + 1 = 23}$ (prime number). The primitive elements of the GF (23) are $(5, 7, 10, 11, 14, 15, 19, 20, 21)$.

For the primitive root $\boldsymbol{x = 5}$, then the initial block $I = \{1,\ 2,\ 4,\ 8,\ 16,\ 9,\ 18,\ 13,\ 3,\ 6,\ 12\}$

For the primitive root $\boldsymbol{x = 7}$, then the initial block $I = \{1,\ 3,\ 9,\ 4,\ 12,\ 13,\ 16,\ 12,\ 6,\ 18,\ 8\}$

For the primitive root $\boldsymbol{x = 10}$, then the initial block $I = \{1,\ 8,\ 18,\ 6,\ 2,\ 16,\ 13,\ 12,\ 4,\ 9,\ 3\}$

For the primitive root $\boldsymbol{x = 11}$, then the initial block $I = \{1,\ 6,\ 13,\ 9,\ 8,\ 2,\ 12,\ 3,\ 18,\ 16,\ 4\}$

For the primitive root $\boldsymbol{x = 14}$, then the initial block $I = \{1,\ 12,\ 6,\ 3,\ 13,\ 18,\ 9,\ 16,\ 8,\ 4,\ 2\}$

For the primitive root $\boldsymbol{x = 15}$, then the initial block $I = \{1,\ 18,\ 2,\ 13,\ 4,\ 3,\ 8,\ 6,\ 16,\ 12,\ 9\}$

For the primitive root $\boldsymbol{x = 19}$, then the initial block $I = \{1,\ 16,\ 3,\ 2,\ 9,\ 6,\ 4,\ 18,\ 12,\ 8,\ 13\}$

For the primitive root $\boldsymbol{x = 20}$, then the initial block $I = \{1,\ 9,\ 12,\ 16,\ 6,\ 8,\ 3,\ 4,\ 13,\ 2,\ 18\}$

For the primitive root $\boldsymbol{x = 21}$, then the initial block $I = \{1,\ 4,\ 16,\ 18,\ 3,\ 12,\ 2,\ 8,\ 9,\ 13,\ 6\}$

It is to be noted here that in all these Initial Blocks, the treatments are same so the design (D) of interest with $v = 23$ can be constructed using any of these initial blocks. Here design is constructed using the initial block (1, 2, 4, 8, 16, 9, 18, 13, 3, 6, 12) obtained from $x = 5$. The remaining blocks of the design are obtained by using the cyclic shift method under $mod(23)$ through initial block.

This design is the symmetric BIB Design with the parameters $v = 23 = b,\ r = 11 = k,\ \lambda = 5$, which can be written as $v = 2t + 1 = b,\ k = t$ with replication of each treatment being $t\&\lambda = t * 1/2$. From this design D the neighbour design is obtained using the method of border plots. The neighbour design D^* thus obtained from D can be written as

$$D^* =$$

$$\begin{array}{c|ccccccccccc|c}
\mathbf{12} & 1 & 2 & 4 & 8 & 16 & 9 & 18 & 13 & 3 & 6 & 12 & \mathbf{1} \\
\mathbf{13} & 2 & 3 & 5 & 9 & 17 & 10 & 19 & 14 & 4 & 7 & 13 & \mathbf{2} \\
\mathbf{14} & 3 & 4 & 6 & 10 & 18 & 11 & 20 & 15 & 5 & 8 & 14 & \mathbf{3} \\
\mathbf{15} & 4 & 5 & 7 & 11 & 19 & 12 & 21 & 16 & 6 & 9 & 15 & \mathbf{4} \\
\mathbf{16} & 5 & 6 & 8 & 12 & 20 & 13 & 22 & 17 & 7 & 10 & 16 & \mathbf{5}
\end{array}$$

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17	6	7	9	13	21	14	23	18	8	11	17	**6**
18	7	8	10	14	22	15	1	19	9	12	18	**7**
19	8	9	11	15	23	16	2	20	10	13	19	**8**
20	9	10	12	16	1	17	3	21	11	14	20	**9**
21	10	11	13	17	2	18	4	22	12	15	21	**10**
22	11	12	14	18	3	19	5	23	13	16	22	**11**
23	12	13	15	19	4	20	6	1	14	17	23	**12**
1	13	14	16	20	5	21	7	2	15	18	1	**13**
2	14	15	17	21	6	22	8	3	16	19	2	**14**
3	15	16	18	22	7	23	9	4	17	20	3	**15**
4	16	17	19	23	8	1	10	5	18	21	4	**16**
5	17	18	20	1	9	2	11	6	19	22	5	**17**
6	18	19	21	2	10	3	12	7	20	23	6	**18**
7	19	20	22	3	11	4	13	8	21	1	7	**19**
8	20	21	23	4	12	5	14	9	22	2	8	**20**
9	21	22	1	5	13	6	15	10	23	3	9	**21**
10	22	23	2	6	14	7	16	11	1	4	10	**22**
11	23	1	3	7	15	8	17	12	2	5	11	**23**

Left and Right Neighbours of a Treatment of the Design

The left & right neighbours of treatments for the neighbour design, when $\boldsymbol{t} = \mathbf{5}$, with parameters $v = 23 = b$, $r = 11 = k$ are summarized in the following table:

Table 2

Other Left Neighbours	Left Neighbours	Treatment Numbers	Right Neighbours	Other Right Neighbours
6, 8, 11, 12, 15, 16, 18	20, 21, 22, 23	1	2, 3, 4, 5	7, 9, 10, 13, 14, 17, 19
7, 9, 12, 13, 16, 17, 19	21, 22, 23, 1	2	3, 4, 5, 6	8, 10, 11, 14, 15, 18, 20
8, 10, 13, 14, 17, 18, 20	22, 23, 1, 2	3	4, 5, 6, 7	9, 11, 12, 15, 16, 19, 21
9, 11, 14, 15, 18, 19, 21	23, 1, 2, 3	4	5, 6, 7, 8	10, 12, 13, 16, 17, 20, 22
10, 12, 15, 16, 19, 20, 22	1, 2, 3, 4	5	6, 7, 8, 9	11, 13, 14, 17, 18, 21, 23
11, 13, 16, 17, 20, 21, 23	2, 3, 4, 5	6	7, 8, 9, 10	12, 14, 15, 18, 19, 22, 1
12, 14, 17, 18, 21, 22, 1	3, 4, 5, 6	7	8, 9, 10, 11	13, 15, 16, 19, 20, 23, 2
13, 15, 18, 19, 22, 23, 2	4, 5, 6, 7	8	9, 10, 11, 12	14, 16, 17, 20, 21, 1, 3
14, 16, 19, 20, 23, 1, 3	5, 6, 7, 8	9	10, 11, 12, 13	15, 17, 18, 21, 22, 2, 4
15, 17, 20, 21, 1, 2, 4	6, 7, 8, 9	10	11, 12, 13, 14	16, 18, 19, 22, 23, 3, 5
16, 18, 21, 22, 2, 3, 5	7, 8, 9, 10	11	12, 13, 14, 15	17, 19, 20, 23, 1, 4, 6
17, 19, 22, 23, 3, 4, 6	8, 9, 10, 11	12	13, 14, 15, 16	18, 20, 21, 1, 2, 5, 7
18, 20, 23, 1, 4, 5, 7	9, 10, 11, 12	13	14, 15, 16, 17	19, 21, 22, 2, 3, 6, 8
19, 21, 1, 2, 5, 6, 8	10, 11, 12, 13	14	15, 16, 17, 18	20, 22, 23, 3, 4, 7, 9
20, 22, 2, 3, 6, 7, 9	11, 12, 13, 14	15	16, 17, 18, 19	21, 23, 1, 4, 5, 8, 10

21, 23, 3, 4, 7, 8, 10	12, 13, 14, 15	16	17, 18, 19, 20	22, 1, 2, 5, 6, 9, 11
22, 1, 4, 5, 8, 9, 11	13, 14, 15, 16	17	18, 19, 20, 21	23, 2, 3, 6, 7, 10, 12
23, 2, 5, 6, 9, 10, 12	14, 15, 16, 17	18	19, 20, 21, 22	1, 3, 4, 7, 8, 11, 13
1, 3, 6, 7, 10, 11, 13	15, 16, 17, 18	19	20, 21, 22, 23	2, 4, 5, 8, 9, 12, 14
2, 4, 7, 8, 11, 12, 14	16, 17, 18, 19	20	21, 22, 23, 1	3, 5, 6, 9, 10, 13, 15
3, 5, 8, 9, 12, 13, 15	17, 18, 19, 20	21	22, 23, 1, 2	4, 6, 7, 10, 11, 14, 16
4, 6, 9, 10, 13, 14, 16	18, 19, 20, 21	22	23, 1, 2, 3	5, 7, 8, 11, 12, 15, 17
5, 7, 10, 11, 14, 15, 17	19, 20, 21, 22	23	1, 2, 3, 4	6, 8, 9, 12, 13, 16, 18

Here also it is observed that each treatment has once all other treatments as either a left neighbour or a right neighbour. So it is a design which is balanced for neighbours with$\boldsymbol{\lambda' = 1}$. The Balanced Incomplete Block Designs are constructed for a number of values of t so that v is a prime number and Neighbour Design for these BIBD are also obtained.

Due to the scarcity of space, it is not possible to illustrate the designs for a various number of treatments here. It has been observed that for each of t (an odd number) obtained in the Neighbour Design each treatments has once all other treatments as either a left neighbour or a right neighbour and $v = b = 2t + 1, r = k = t$ and $\lambda = t - 1/2$ and $\lambda' = 1$. Hence the design obtained is a Balanced Incomplete Block Design and the Neighbour Designs

3. Conclusion:

Using the initial block proposed in this paper designs can be constructed with parameters $\boldsymbol{v = 2t + 1 = b}$, $\boldsymbol{r = t = k}$, $\boldsymbol{\lambda = \frac{t-1}{2}}$, where (**$t$ ≥1**) odd positive integer number. The constructed design is balanced for neighbours with $\boldsymbol{\lambda' = 1}$.

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