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Approximation of Generalized Compound Ray leigh Distribution Parameters Through Bayesian with General Entropy

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Abstract

This paper considers the Bayes estimate of Location parameter α of three parameter Generalized Compound Rayleigh distribution with a chaotic loss known as General Entropy loss function (GELF). The Lindley approximation is applied to obtain the Approximate Bayes estimate of Location parameter of Generalized Compound Rayleigh distribution under the GELF. The obtained results are compared through R-programming.

Keywords: Bayes estimate, R- Programming, Approximation, Lindley Technique, Generalized Compound Rayleigh distribution, General Entropy loss function.

1. INTRODUCTION

The pdf Generalized Compound Rayleigh Distribution is given by

$$f(x;\alpha,\beta,\gamma)=\frac{\alpha}{\gamma}\beta^{\frac{1}{\gamma}}x^{(\alpha-1)}(\beta+x^{\alpha})^{-(\gamma+1)}; \qquad x,\alpha,\beta,\gamma>0(1.1)$$

as special case of the three-parameter Burr type XII distribution Dubey(1968)

Distribution Function is given by.

$$\mathrm{F(x)}=1-(1-\beta x^{\alpha})^{-\frac{1}{\gamma}}; \qquad x,\alpha,\beta,\gamma>0(1.2)$$

The symmetrical Loss function associates the equal importance to the losses due to overestimation and under estimation with equal magnitudes however in some estimation problems such an assumption may be inappropriate. Overestimation may be more serious than underestimation or Vice-versa Ferguson (1967).

In many practical situations, it appears to be more realistic to express the loss in terms of the ration $\frac{\hat{\theta}}{\theta}$. In this case Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the Entropy loss

$$L(\delta)\ \alpha\ [\delta^{P}-p\ log_e(\delta)-1 \quad ;\text{Where } \delta=\frac{\hat{\theta}}{\theta}(1.3)$$

And whose minimum occurs at $\hat{\theta}=\theta$ where p>0, a positive error ($\hat{\theta}>\theta$) causes more serious consequences that a negative error and vice-versa. For small |p| value the function is almost symmetric, when both $\hat{\theta}$ and θ are measured in a logarithmic scale and approximately.

$$L(\delta)=b[\delta-log_e(\delta)-1]; \quad b>0; \qquad \text{Where } \delta=\frac{\hat{\theta}}{\theta}(1.4)$$

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2.TheEstimators

Let$x_1 \leq x_2 \leq \ldots\ldots\ldots \leq x_n$be the n failures in complete sample case. Thelikelihood function is given by

$$L(\underline{x}|\alpha,\beta,\gamma) = \frac{\alpha^n}{\gamma^n}\beta^{n/\gamma}\prod_{j=1}^{n} x_j^{\alpha-1}\prod_{j=1}^{n}(\beta + x_j\alpha)^{-\left(\frac{1}{\gamma}+1\right)} \quad (2.1)$$

$$L(\underline{x}|\alpha,\beta,\gamma) = \left(\frac{\alpha}{\gamma}\right)^n U e^{-T/\gamma} \quad (2.2)$$

Where

$$T = \sum_{j=1}^{n} log\left[1 + \frac{x_j^{\alpha}}{\beta}\right] \text{and} U = \prod_{j=1}^{n} \frac{x_j^{\alpha-1}}{\beta + x_j^{\alpha}}$$

from equation(2.1),the log likelihood function is

$$Log\ L = n\ log\alpha + \frac{n}{\gamma}log\beta - n\ log\gamma + (\alpha-1)\sum_{j=1}^{n} logx_j - \left(\frac{1}{\gamma} + 1\right)\sum_{j=1}^{n} log(\beta + x_j^{\alpha}) \quad (2.3)$$

The maximum likelihood estimators (MLE) of the parameters namely $\hat{\alpha}_{MLE}$, $\hat{\beta}_{MLE}$ and $\hat{\gamma}_{MLE}$which can be obtain by differentiating equation(2.3) with respect to α, β and γ and solving by Newton Raphson Method.

3.Bayesian Approximation ofunknown Locationparameterα (Lindley(1980), Solimon (2001)

The Joint prior density of the parameters α, β, γ is given by

$$G(\alpha,\beta,\gamma) = g_1(\alpha)g_2(\beta)g_3(\gamma|\beta) \quad (3.1)$$

where

$$g_1(\alpha) = c \quad (3.2)$$

$$g_2(\beta) = \frac{1}{\delta}e^{-\frac{\beta}{\delta}} \quad (3.3)$$

$$g_3(\gamma) = \frac{1}{\Gamma\xi}\beta^{-\xi}\gamma^{\xi+1}e^{-\frac{\gamma}{\beta}} \quad (3.4)$$

The Joint posterior combing the likelihood equation(2.2) and joint prior equation(3.1) is

$$h^*(\alpha,\beta,\gamma|\underline{x}) = \frac{\beta^{-\xi}\gamma^{\xi+1}exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)}{\int_\alpha\int_\beta\int_\gamma \beta^{-\xi}\gamma^{\xi+1}exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (3.5)$$

The Approximate Bayes Estimator is given by

$$\cup(\Theta) = \cup(\alpha,\beta,\gamma) \quad (3.6)$$

$$\hat{\cup}_{BS} = E(\cup|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma \cup(\alpha,\beta,\gamma)G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma} \quad (3.7)$$

Approximation Through Lindley

Lindley Approximation Procedure

The Bayes estimators of a function $\mu = \mu(\theta, p)$ of the unknown parameter θ and p under squared error loss is the posterior mean

$$\hat{\mu}_{BS} = E(\mu|\underline{x}) = \frac{\iint \mu(\theta,p)h^*(\theta,p|\underline{x})d\theta dp}{\iint h^*(\theta,p|\underline{x})d\theta dp} \quad (3.7a)$$

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The ratio of integrals in equation (3.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$E\left(\mu(\theta,p)|\underline{x}\right)=\frac{\int \mu(\theta).e^{(l(\theta)+\rho(\theta))}d\theta}{\int e^{(l(\theta)+\rho(\theta))}.d\theta}$ (3.7b)

Lindley developed approximate procedure for evaluation of posterior expectation of$\mu(\theta)$. (see Sinha(1986),Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\mu(\theta)$ in equation(3.7a and 3.7b) under SELF, where where$\rho(\theta)=\log g(\theta)$, and $g(\theta)$ is an arbitrary function of θ and $l(\theta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (3.7) is given by

$E\left(\cup(\alpha,\beta,\gamma|\underline{x}\right)=\cup(\Theta)+\frac{1}{2}[A(\cup_1\sigma_{11}+\cup_2\sigma_{12}+\cup_3\sigma_{13})+B(\cup_1\sigma_{21}+\cup_2\sigma_{22}+\cup_3\sigma_{23})+P(\cup_1\sigma_{31}+\cup_2\sigma_{32}+\cup_3\sigma_{33})]+(\cup_1 a_1+\cup_2 a_2+\cup_3 a_3+a_4+a_5)+0\left(\frac{1}{n^2}\right)$(3.8)Above equation is evaluated at MLE = $(\hat{\alpha},\hat{\beta},\hat{\gamma})$

where

$a_1=\rho_1\sigma_{11}+\rho_2\sigma_{12}+\rho_3\sigma_{13}$ (3.9)

$a_2=\rho_1\sigma_{21}+\rho_2\sigma_{22}+\rho_3\sigma_{23}$ (3.10)

$a_3=\rho_1\sigma_{31}+\rho_2\sigma_{32}+\rho_3\sigma_{33}$(3.11)

$a_4=\cup_{12}\sigma_{12}+\cup_{13}\sigma_{13}+\cup_{23}\sigma_{23}$ (3.12)

$a_5=\frac{1}{2}(\cup_{11}\sigma_{11}+\cup_{22}\sigma_{22}+\cup_{33}\sigma_{33})$; (3.13)

And

$A=[\sigma_{11}l_{111}+2\sigma_{12}l_{121}+2\sigma_{13}l_{131}+2\sigma_{23}l_{231}+\sigma_{22}l_{221}+\sigma_{33}l_{331}]$(3.14)

$B=[\sigma_{11}l_{112}+2\sigma_{12}l_{122}+2\sigma_{13}l_{132}+2\sigma_{23}l_{232}+\sigma_{22}l_{222}+\sigma_{33}l_{332}]$(3.15)

$P=[\sigma_{11}l_{113}+2\sigma_{13}l_{133}+2\sigma_{12}l_{123}+2\sigma_{23}l_{233}+\sigma_{22}l_{223}+\sigma_{33}l_{333}]$(3.16)

To apply Lindley approximation on equation (3.8), we first obtain

$\sigma_{ij}=\left[-l_{ijk}\right]^{-1}i,j,k=1,2,3$

Likelihood function from equation (3.2) is

$L=\frac{\alpha^n}{\gamma^n}\beta^{\frac{n}{\gamma}}\prod_{j=1}^{n}x_j^{\alpha-1}\prod_{j=1}^{n}\left(\beta+x_j{}^{\alpha}\right)^{-\left(\frac{1}{\gamma}+1\right)}$; $(x,\alpha,,\gamma>0)$

$Log\ L=n\ log\alpha-nlog\gamma+\frac{n}{\gamma}log\beta+(\alpha-1)\sum_{j=1}^{n}logx_j-\left(\frac{1}{\gamma}+1\right)\sum_{j=1}^{n}\frac{x_j{}^{\alpha}logx_j}{\beta+x_j{}^{\alpha}}$(3.17)

Nowl_{ijk}'s are the partial derivatives of α,β,γ respectively of equation(3.17), given as

$l_{111}=\frac{2n}{\alpha^3}-\left(\frac{1}{\gamma}+1\right)\omega_{133}$where$\omega_{133}=\sum\frac{x_j{}^{\alpha}(\beta-x_j{}^{\alpha})(\log x_j)^3}{(\beta+x_j{}^{\alpha})^3}$ (3.18) $l_{222}=\frac{2n}{\gamma\beta^3}-2\left(\frac{1}{\gamma}+1\right)\delta_{13}$where $\delta_{13}=\sum_{j=1}^{n}\frac{1}{(\beta+x_j)^3}$(3.19)

$l_{333}=-\frac{2n}{\gamma^3}-\frac{6n\ log\beta}{\gamma^4}+\frac{6}{\gamma^4}\delta_{10}$ where$\delta_{10}=\sum_{j=1}^{n}log\left(\beta+x_j{}^{\alpha}\right)$(3.20)

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$l_{112} = \left(\frac{1}{\gamma} + 1\right) \omega_{123}$where$\omega_{123} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha}(\beta - x_j{}^{\alpha})(\log x_j)^2}{(\beta + x_j{}^{\alpha})^3}$(3.21)

and $l_{112} = l_{121}$

$l_{113} = \frac{\beta}{\gamma^2} \omega_{122}$ where $\omega_{122} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha}(log x_j)^2}{(\beta + x_j{}^{\alpha})^2}$ (3.22)

$l_{221} = -2\left(\frac{1}{\gamma} + 1\right) \omega_{113}$ where $\omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha} log x_j}{(\beta + x_j{}^{\alpha})^3}$(3.23)

$l_{221} = l_{212}$

$l_{223} = \frac{n}{(\gamma\beta)^2} - \frac{1}{(\gamma)^2} \delta_{12}$where$\delta_{12} = \sum_{j=1}^{n} \frac{1}{(\beta + x_j{}^{\alpha})^2}$ (3.24)

$l_{331} = -\frac{2}{\gamma^3} \omega_{11}$ where $\omega_{11} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha} log x_j}{(\beta + x_j{}^{\alpha})}$ (3.25)

$l_{331} = l_{313}$

$l_{332} = \frac{\partial}{\partial\gamma}\left(\frac{\partial^2 L}{\partial\gamma\partial\beta}\right) = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)$ (3.26)

$l_{332} = l_{323}$

$l_{231} - \frac{\omega_{14}}{\gamma^2}$ (3.27)

$l_{231} = l_{213}$

$l_{123} = -\frac{\omega_{14}}{\gamma^2}$ (3.28)

$l_{123} = l_{132}$

$l_{133} = \frac{-2}{\gamma^2} \sum_{j=1}^{n} \frac{x_j{}^{\alpha} \log x_j}{(\beta + x_j{}^{\alpha})} = -\frac{2}{\gamma^2} \omega_{11}$(3.29)

$l_{122} = -2\left(\frac{1}{\gamma} + 1\right) \omega_{113}$where$\omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\alpha} \log x_j}{(\beta + x_j{}^{\alpha})^3}$ (3.30)

$l_{233} = \frac{2n}{\beta\gamma^3} - \frac{2}{\gamma^3} \sum_{j=1}^{n} \frac{1}{\beta + x_j{}^{\alpha}} = \frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)$ (3.31)

The matrix of derivatives is given as

$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix}$ (3.32)

$$= \begin{bmatrix} \frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right) \omega_{133} & , \left(\frac{1}{\gamma} + 1\right) \omega_{123} & -\frac{\beta}{\gamma^2} \omega_{122} \\ -2\left(\frac{1}{\gamma} + 1\right) \omega_{113} & , \frac{2n\gamma}{\gamma\beta^3} - 2\left(\frac{1}{\gamma} + 1\right) \delta_{13} & \frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2} \delta_{12} \\ \frac{-2}{\gamma^3} \omega_{11} & , \frac{-2}{\gamma^3}\left(\frac{n}{\gamma} - \delta_{11}\right) & , -\frac{2n}{\gamma^3} - \frac{6n \, log\beta}{\gamma^4} + \frac{6}{\gamma^4} \delta_{10} \end{bmatrix}$$

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$$[-l_{ijk}] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$Determinant\ of\ \ [-l_{ijk}], D = \{M_{11}[M_{22}M_{33} - M_{23}M_{32}] - M_{12}[M_{21}M_{33} - M_{31}M_{23}] + M_{13}[M_{21}M_{32} - M_{22}M_{33}]\}$(3.33)

$$[-l_{ijk}]^{-1} = \frac{(Adjoint\ of\ [-l_{ijk}])'}{D}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \frac{Y_{11}}{D} & \frac{Y_{12}}{D} & \frac{Y_{13}}{D} \\ \frac{Y_{21}}{D} & \frac{Y_{22}}{D} & \frac{Y_{23}}{D} \\ \frac{Y_{31}}{D} & \frac{Y_{32}}{D} & \frac{Y_{33}}{D} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}; (3.34)$$

Approximate Bayes Estimator

$\cup\ (\alpha, \beta, \gamma) = \cup$

$\hat{\cup}_{AB} = E(\cup\ |\underline{x})$

evaluated from equation number and from joint prior density, we have

$G(\alpha, \beta, \gamma) = g(\alpha)g_2(\beta)g_3(\gamma|\beta)$

$\rho = \log G = \log C - log\delta - log\lceil\xi + (\xi - 1)log\gamma - \xi log\beta$

$-\left(\frac{\gamma}{\beta} + \frac{\beta}{\delta}\right)$ (3.35)

logG$= constant - \xi log\beta + (\xi - 1)log\gamma - \frac{\gamma}{\beta} - \frac{\beta}{\delta}$

$\rho_1 = \frac{\delta\rho}{\delta\alpha} = 0$(3.36)

$\rho_2 = \frac{\delta\rho}{\delta\beta} = \frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}$(3.37)

$\rho_3 = \frac{\delta\rho}{\delta\gamma} = \frac{\xi - 1}{\gamma} - \frac{1}{\beta}$(3.38)

Using equation(3.14) to equation(3.33), we have

$A = \frac{1}{D}\left[Y_{11}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133}\right) + 2Y_{12}\left(\frac{1}{\gamma} + 1\right)\omega_{123} + 2Y_{13}\frac{\beta}{\gamma^3}\omega_{122} - 2Y_{23}\frac{\omega_{14}}{\gamma^2} - 2Y_{22}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2}{\gamma^3}Y_{33}\omega_{11}\right]$(3.39)

$B = \frac{1}{D}\left[\left(\frac{1}{\gamma} + 1\right)\omega_{123}Y_{11} - 4Y_{12}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - 2Y_{13}\left(-\frac{\omega_{14}}{\gamma^2}\right) + (Y_{22} + 2Y_{23})\left(\frac{n}{(\gamma\beta)^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right]$
(3.40)

$P = \frac{1}{D}\left[\frac{Y_{11}\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}\omega_{14}}{\gamma^4} - \frac{4Y_{13}\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right) + Y_{22}\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + Y_{33}\left(-\frac{2n}{\gamma^3} - \frac{6n\ log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)\right]$(3.41)

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Now

$\hat{\cup}_{AB} = E(\cup\,|\underline{x})$

$$E(\cup\,|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_{4+}a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right)$$

$E(\cup\,|\underline{x}) = U + \varphi_1 + \varphi_2$ (3.42)

where

$\varphi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5$ (3.43)

$\varphi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).U_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).U_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})U_3]$(3.44)

evaluated at the MLE$\hat{\cup} = (\hat{\alpha}, \hat{\beta}, \hat{\gamma})$where

$a_1 = 0.\sigma_{11} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D}$(3.45)

$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23}$

$a_2 = 0.\sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D}$(3.46)

$a_3 = 0.\sigma_{31} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{32}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{33}}{D}$ (3.47)

$a_4 = \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23}$(3.48)

$a_5 = \frac{1}{2D}(Y_{11}U_{11} + Y_{22}U_{22} + Y_{33}U_{33})$ (3.49)

3.Approximate Bayes Estimator under General Entropy loss function(GELF)

$\hat{U}_{ABE} = \left[E_h\left(\frac{1}{\theta}\right)\right]^{-1}$(3.50)

Where;

$E_u(\theta|\underline{x}) = \frac{\int_\alpha\int_\beta\int_\gamma\frac{1}{\theta}G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}{\int_\alpha\int_\beta\int_\gamma G^*(\alpha,\beta,\gamma)d\alpha d\beta d\gamma}$(3.51)

The equation(3.50)is evaluated by method of Lindley approximation by replacingθin$\cup\ (\alpha, \beta, \gamma)$

Special cases–

Approximate Bayes Estimate of α

$\cup\ (\alpha, \beta, \gamma) = \cup = \frac{1}{\alpha}$

$E\left(\frac{1}{\alpha}|\underline{x}\right) = \frac{1}{\alpha} + \varphi_1 + \varphi_2;$ (3.52)

$\cup_1 = \frac{\partial}{\partial\alpha}\left(\frac{1}{\alpha}\right) = -\frac{1}{\alpha^2}$; $\cup_{11} = \frac{\partial}{\partial\alpha}\left(-\frac{1}{\alpha^2}\right) = \frac{2}{\alpha^3}$; $\cup_{12} = \cup_{13} = 0$

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$\cup_2 = \cup_{21} = \cup_{22} = \cup_{23} = 0$

$\cup_3 = \cup_{31} = \cup_{32} = \cup_{33} = 0$

$E\left(\frac{1}{\alpha}|\underline{x}\right) = U + \varphi_1 + \varphi_2$

$\varphi_1 = -\frac{1}{\alpha^2}\left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} + \frac{-1}{\beta}\right)\frac{Y_{13}}{D}\right] + \frac{Y_{13}}{D} - \frac{Y_{11}}{\alpha^3}$(3.53)

$\varphi_2 = -\frac{1}{2\alpha^2}(A\sigma_{11} + B\sigma_{21} + P\sigma_{31})$(3.54)

$\hat{\alpha}_{ABE} = \alpha[1 - \alpha^{-1}\Delta_4]^{-1};$ at $\left(\hat{\alpha}_{ML}, \hat{\beta}_{ML}, \hat{\gamma}_{ML}\right)$ (3.55)

where;

$\Delta_4 = \left[\left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta^2} - \frac{1}{\delta}\right)\frac{Y_{12}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{13}}{D} - .\frac{Y_{11}}{2D}\right] + \frac{1}{2}\left[\frac{Y_{11}}{D}\left(\frac{2n}{\alpha^3} - \left(\frac{1}{\gamma} + 1\right)\omega_{133} + 2\frac{Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{123} + 2\frac{Y_{13}}{D}\frac{\beta}{\gamma^2}\omega_{122} - 2\frac{Y_{23}}{D}\frac{\omega_{14}}{\gamma^2} - 2\frac{Y_{22}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2}{\gamma^3}\frac{Y_{33}}{D}\omega_{11}\right)\right]\frac{Y_{11}}{D} + \frac{Y_{21}}{2D}\left[\frac{Y_{11}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{123} - \frac{4Y_{12}}{D}\left(\frac{1}{\gamma} + 1\right)\omega_{113} - \frac{2Y_{13}}{D}\frac{\omega_{14}}{\gamma^2} + \left(\frac{Y_{22}+2Y_{23}}{D}\right).\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \frac{Y_{33}}{D}\left(\frac{2}{\gamma^3}\left(\frac{n}{\beta} - \delta_{11}\right)\right)\right] + \frac{Y_{31}}{D}\left[\frac{Y_{11}}{D}\frac{\beta}{\gamma^2}\omega_{122} - \frac{2Y_{12}}{D}\frac{\omega_{14}}{\gamma^4} - 4\frac{Y_{13}}{D}\frac{\omega_{11}}{\gamma^3} + \frac{4Y_{23}}{D\gamma^3} + \left(\frac{n}{\beta} - \delta_{11}\right) + \frac{Y_{22}}{D}\left(\frac{n}{\gamma^2\beta^2} - \frac{1}{\gamma^2}\delta_{12}\right) + \frac{Y_{33}}{D}\left(\frac{-2n}{\gamma^3} - \frac{6n\log\beta}{\gamma^4} + \frac{6}{\gamma^4}\delta_{10}\right)\right]$(3.56)

Numerical Comparison

The simulations and numerical calculations are done by using R Language programming and results are presented in form of tables in table (1).

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters α, β, γ, taken as $\alpha = 0.85$, $\beta = 0.71$ and $\gamma = 0.95$ equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimator ,and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the table from (1), and parameters of prior distribution a =3 and b =4.

3. Table (1) also presents the MLE of parameter of γ (for known α *and* β) and Approximate Bayes estimator under GELF (for α *unknown*) and their respective MSE's.

Table (1)

Mean and MSE'S of α

($\alpha = 0.85$, $\beta = 0.71$ and $\gamma = 0.95$)

n	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**
$\hat{\alpha}_{ML}$	0.7546328	0.7589642	0.8125469	0.8645712	0.9451287	0.9998756	1.3000043	1.9000032
	[0.152134]	**[0.126537]**	**[0.098745]**	**[0.007451]**	**[0.004213]**	**[0.004154]**	**[0.000231]**	**[0.000321]**
$\hat{\alpha}_{ABE}$	0.7789877	0.8007956	0.8123467	0.8897454	0.9710110	0.9978542	0.9888757	1.0121435
	[7.15e-04]	**[8.23e-04]**	**[8.24e-04]**	**[0.001415]**	**[0.001725]**	**[0.001124]**	**[0.004221]**	**[0.004222]**

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