

The relation among vague filters and Residuated Lattices

R.Illakiya¹, S.Muhamead Sanawaz², SenthamilSelvi³, Kavitha⁴, Rajalakshmi⁵ Shreedharan M.D⁶
and Jeganathan M⁷

¹⁻⁵Nehru Institute of Technology, Department of Mathematics, Coimbatore.

⁶Associate Professor, Excel College of Architecture & Planning, Komarapalayam, Tamil Nadu.

⁷Assistant Professor, Department of Environment and Herbal Sciences, Tamil University,
Thanjavur, Tamil Nadu.

nitkavitha@nehrucolleges.com jegann1978@gmail.com

Abstract:

The focus of this paper is to develop The relation between the vague filters and residuated lattices, and its essential properties are investigated. Characterizations of vague filters in residuated lattice are established. We discuss some properties of vague filters in terms of its level subsets. Also the notion of Extended pair of vague filter is introduced and characterize their properties.

1.Introduction:

The notion of residuated lattices is initiated in order to provide a reliable logical foundation for uncertain information processing theory and establish a logical system with truth value in a relatively general lattice. The concept of fuzzy set was introduced by Zadeh (1965) [19]. Since then this idea has been applied to other algebraic structures. Since the fuzzy set is single function, it cannot express the evidence of supporting and opposing. Hence the concept of vague set [6] is introduced in 1993 by W.L.Gau and Buehrer. D.J. In a vague set A, there are two membership functions: a truth membership function t_A and a false membership function f_A , where t_A and f_A are lower bound of the grade of membership respectively and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership in a vague set A is a subinterval $[t_A(x), 1-f_A(x)]$ of $[0, 1]$. Vague set is an extension of fuzzy sets. The idea of vague sets is that the membership of every elements which can be divided into two aspects including supporting and opposing. (Manikandan et.al., 2016, Sethuraman et.al., 2016, Senthil Thambi et.al., 2016, Ashok et.al., 2018, Senthilkumar et.al., 2018, Sundar and Jeganathan 2019 & 2020, Anandan et.al., 2019, Murugavel et.al., 2019, Arokiaswamy et.al., 2019 & 2020, Ganesh Babu et.al., 2020, Gomathi et.al., 2019 & 2020, Manju et.al., 2020, Leema Rose et.al., 2020). With the development of vague set theory, some structure of algebras corresponding to vague set have been studied. R.Biswas [3] initiated the study of vague algebras by studying vague groups. T.Eswarlal [5] study the vague ideals and normal vague ideals in semirings. H.Hkam, etc [13] study the vague relations and its properties. Quotient algebras are basic tool for exploring the structures of algebras. There are close correlations among filters, congruences and quotient algebras. (Vasanthi and Jeganathan 2007, Vasanthi et.al., 2008, Raajasubramanian et.al., 2011, Jeganathan et.al., 2012, 2014, 2020 & 2021, Sridhar et.al., 2012, Gunaselvi et.al., 2014 & 2020, Premalatha et.al., 2015, Seshadri et.al., 2015, Shakila et.al., 2015, Ashok et.al., 2016, Satheesh Kumar et.al., 2016 & 2019).

2.Vague Filters on residuated Lattice

1. $V_A(I) \geq V_A(x)$
2. $V_A(y) \geq \min(V_A(x \rightarrow y), V_A(x))$

Definition 2.1:

A Vague set A of L is called a vague filter of L, if for any $x, y \in L$:

Theorem 2.2:

Let A be a vague filter of L. Then, for any $x, y \in L$ if $x \leq y$, then $V_A(x) \leq V_A(y)$.

Proof:

Since $x \leq y$, it follows that $x \rightarrow y = I$. Since A is a vague filter of L , we have

$$V_A(y) \geq \min(V_A(x \rightarrow y), V_A(x)) \text{ and } V_A(I) \geq V_A(x) \text{ for any } x, y \in L.$$

$$\begin{aligned} \text{Therefore } V_A(y) &\geq \min(V_A(x \rightarrow y), V_A(x)) \\ &= \min(V_A(I), V_A(x)) \geq \min(V_A(x), V_A(x)) = V_A(x). \end{aligned}$$

$$\text{Therefore } V_A(x) \leq V_A(y).$$

Theorem 2.3:

Let A be a vague set on L . Then A is vague filter of L , if and only if, for any $x, y, z \in L$ $V_A(I) \geq V_A(x)$ and $V_A(x \rightarrow z) \geq \min(V_A(y \rightarrow (x \rightarrow z)), V_A(y))$

Proof:

Let A be vague filter of L , obviously $V_A(I) \geq V_A(x)$ and $V_A(I) \geq V_A(x)$ and $V_A(x \rightarrow z) \geq \min(V_A(y \rightarrow (x \rightarrow z)), V_A(y))$ holds for any $x, y, z \in L$. Taking $x = I$ in $V_A(x \rightarrow z) \geq \min(V_A(y \rightarrow (x \rightarrow z)), V_A(y))$, we have $V_A(z) = V_A(I \rightarrow z) \geq \min(V_A(y \rightarrow (I \rightarrow z)), V_A(y)) = \min(V_A(y \rightarrow z), V_A(y))$.

Remark 2.5:

A vague set on L is a vague filter of L , if and only if, for any $x, y, z \in L$:if $x \rightarrow (y \rightarrow z) = I$ then $V_A(z) \geq \min(V_A(x), V_A(y))$.

Remark 2.6:

A vague set on L is a vague filter of L , if and only if, for any $x, y, z \in L$:
 if $a_n \rightarrow (a_{n-1} \rightarrow \dots \rightarrow (a_1 \rightarrow x) \dots) = I$, then $V_A(x) \geq \min(V_A(a_n), \dots, V_A(a_1))$

Theorem 2.7:

A vague set on L is a vague filter of L , if and only if, for any $x, y, z \in L$, A satisfies Remark 2.5 and $V_A((x \rightarrow (y \rightarrow z)) \rightarrow z) \geq \min(V_A(x), V_A(y))$.

Proof:

Since $V_A(I) \geq V_A(x)$ holds, and so A is a vague filter of L .

Theorem 2.4:

Let A be a vague set on L . Then A is a vague filter of L , if and only if, for any $x, y, z \in L$, A satisfies if $x \leq y$, then $V_A(x) \leq V_A(y)$ for any $x, y \in L$ and $V_A(x * y) \geq \min(V_A(x), V_A(y))$.

Proof:

Assume that A is a vague filter of L , obviously if $x \leq y$, then $V_A(x) \leq V_A(y)$ holds for any $x, y \in L$. Since $x \leq y \rightarrow (x * y)$, we have $V_A(y \rightarrow (x * y)) \geq V_A(x)$.

By Definition 2.1 (2),

$$\text{it follows that } V_A(x * y) \geq \min(V_A(y), V_A(y \rightarrow (x * y))) \geq \min(V_A(y), V_A(x)).$$

Conversely, assume that if $x \leq y$, then $V_A(x) \leq V_A(y)$ and $V_A(x * y) \geq \min(V_A(x), V_A(y))$ holds. Taking $y = I$, we get $V_A(I) \geq V_A(x)$.

$$\text{As } x * (x \rightarrow y) \leq y, \text{ thus } V_A(y) \geq V_A(x * (x \rightarrow y)).$$

Therefore $V_A(y) \geq \min(V_A(x), V_A(x \rightarrow y))$.

Hence A is a vague filter of L .

If A is a vague filter of L then Remark 2.5 holds. Since $V_A((x \rightarrow (y \rightarrow z)) \rightarrow z) \geq \min(V_A((x \rightarrow (y \rightarrow z)) \rightarrow (y \rightarrow z)), V_A(y))$.

As $(x \rightarrow (y \rightarrow z)) \rightarrow (y \rightarrow z) = x \vee (y \rightarrow z) \geq x$, by Theorem 2.2 we have $V_A((x \rightarrow (y \rightarrow z)) \rightarrow (y \rightarrow z)) \geq V_A(x)$.

Therefore, $V_A((x \rightarrow (y \rightarrow z)) \rightarrow z) \geq \min(V_A(x), V_A(y))$.

Conversely, suppose $V_A((x \rightarrow (y \rightarrow z)) \rightarrow z) \geq \min(V_A(x), V_A(y))$ is valid. Since $V_A(y) = V_A(I \rightarrow y) = V_A(((x \rightarrow y) \rightarrow (x \rightarrow y)) \rightarrow y) \geq \min(V_A(x \rightarrow y), V_A(x))$.

Hence by Definition

2.1, A is a vague filter of L .

Theorem 2.8:

Let A be a vague set on L . Then A is a vague filter of L , for any $x, y, z \in L$, A satisfies Definition 2.1(1) and $V_A(x \rightarrow z) \geq \min(V_A(x \rightarrow y), V_A(y \rightarrow z))$.

Proof:

Assume that A is vague filter of L. Since $(x \rightarrow y) \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$, it follows from Theorem 2.2 that $V_A((y \rightarrow z) \rightarrow (x \rightarrow z)) \geq V_A(x \rightarrow y)$. As A is a vague filter, so $V_A(x \rightarrow z) \geq \min(V_A(y \rightarrow z), V_A((y \rightarrow z) \rightarrow (x \rightarrow z)))$. We have $V_A(x \rightarrow z) \geq \min(V_A(y \rightarrow z), V_A(x \rightarrow z))$.

Conversely, if $V_A(x \rightarrow z) \geq \min(V_A(x \rightarrow y), V_A(y \rightarrow z))$ for any $x, y, z \in L$, then $V_A(I \rightarrow z) \geq \min(V_A(I \rightarrow y), V_A(y \rightarrow z))$ that is $V_A(z) \geq \min(V_A(y), V_A(y \rightarrow z))$. Hence by definition 2.1 A is a vague filter of L.

Theorem 2.9:

Let A be a vague set on L. Then A is a vague filter of L, if and only if, for any $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, the sets $U(t_A, \alpha)$ ($\neq \emptyset$) and $L(1-f_A, \beta)$ ($\neq \emptyset$) are filters of L, where $U(t_A, \alpha) = \{x \in L / t_A(x) \geq \alpha\}$, $L(1-f_A(x), \beta) = \{x \in L / 1-f_A(x) \geq \beta\}$.

Proof:

Assume A is a vague filter of L, then $V_A(I) \geq V_A(x)$. By the condition $U(t_A, \alpha) \neq \emptyset$, it follows that there exist $a \in L$ such that $t_A(a) \geq \alpha$, and so $t_A(I) \geq \alpha$, hence $I \in U(t_A, \alpha)$. Let $x, x \rightarrow y \in U(t_A, \alpha)$, then $t_A(x) \geq \alpha$, $t_A(x \rightarrow y) \geq \alpha$. Since A is a filter of L, then $t_A(y) \geq \min(t_A(x), t_A(x \rightarrow y)) \geq \min(\alpha, \alpha) = \alpha$.

Hence $y \in U(t_A, \alpha)$. Therefore $U(t_A, \alpha)$ is a filter of L. We will show that $L(1-f_A(x), \beta)$ is a filter of L. Since A is a vague filter of L, then $1-f_A(I) \geq 1-f_A(x)$. By the condition $L(1-f_A(x), \beta) \neq \emptyset$, it follows that there exist $a \in L$ such that $1-f_A(a) \geq \beta$. Therefore we have $1-f_A(I) \geq 1-f_A(a) \geq \beta$. Hence $I \in L(1-f_A(x), \beta)$. Let $x, x \rightarrow y \in L(1-f_A(x), \beta)$, then $1-f_A(x) \geq \beta$, $1-f_A(x \rightarrow y) \geq \beta$. Since A is a vague filter of L, then

$1-f_A(y) \geq \min(1-f_A(x), 1-f_A(x \rightarrow y)) \geq \min(\beta, \beta) = \beta$. It follows that $1-f_A(y) \geq \beta$, hence $y \in L(1-f_A(x), \beta)$. Therefore $L(1-f_A(x), \beta)$ is a filter of L. Conversely, suppose that $U(t_A, \alpha) \neq \emptyset$ and $L(1-f_A(x), \beta) \neq \emptyset$ are filters of L, then, for any $x \in L$, $x \in U(t_A, t_A(x))$ and $x \in L(1-f_A, 1-f_A(x))$.

By $U(t_A, t_A(x)) \neq \emptyset$ and $L(1-f_A, 1-f_A(x)) \neq \emptyset$ are filters of L, it follows that $I \in U(t_A, t_A(x))$ and $I \in L(1-f_A, 1-f_A(x))$, and so $V_A(I) \geq V_A(x)$. For any $x, y \in L$, let $\alpha = \min(t_A(x), t_A(x \rightarrow y))$ and $\beta = \min(1-f_A(x), 1-f_A(x \rightarrow y))$, then $x, x \rightarrow y \in U(t_A, \alpha)$ and $x, x \rightarrow y \in L(1-f_A, \beta)$. And so $y \in U(t_A, \alpha)$ and $y \in L(1-f_A(x), \beta)$.

Therefore $t_A(y) \geq \alpha = \min(t_A(x), t_A(x \rightarrow y))$ and $1-f_A(y) \geq \beta = \min(1-f_A(x), 1-f_A(x \rightarrow y))$. From theorem 3.2, we have A is a vague filter of L.

Theorem 2.10:

Let A, B be two vague filters of L, then $A \cap B$ is also a vague filter of L.

Proof:

Let $x, y, z \in L$ such that $z \leq x \rightarrow y$, then $z \rightarrow (x \rightarrow y) = I$. Since A, B be two vague filters of L, we have $V_A(y) \geq \min(V_A(z), V_A(x))$ and $V_B(y) \geq \min(V_B(z), V_B(x))$. Since $V_{A \cap B}(y) = \min(V_A(y), V_B(y)) \geq \min(\min(V_A(z), V_A(x)), \min(V_B(z), V_B(x))) = \min(\min(V_A(z), V_B(z)), \min(V_A(x), V_B(x))) = \min(V_{A \cap B}(z), V_{A \cap B}(x))$. Since A, B be two vague filters of L, we have $V_A(I) \geq V_A(x)$ and $V_B(I) \geq V_B(x)$. Hence $V_{A \cap B}(I) = \min(V_A(I), V_B(I)) \geq \min(V_A(x), V_B(x)) = V_{A \cap B}(x)$. Then $A \cap B$ is a vague filters of L.

Remark 2.11:

Let A_i be a family of vague sets on L, where i is an index set. Denoting C by the

intersection of A_i , i.e. $\bigcap_{i \in I} A_i$, where $V_C(x) = \min(V_{A_1}(x), V_{A_2}(x), \dots)$ for any $x \in L$.

Note 2.12:

Let A_i be a family of vague filters of L, where $i \in I$, I is an index set, then $\bigcap_{i \in I} A_i$ is also a vague filters of L.

Theorem 2.13:

Let A be a vague set on L. Then

- a. For any $\alpha, \beta \in [0, 1]$, if $A_{(\alpha, \beta)}$ is a filter of L. Then, for any $x, y, z \in L$, $V_A(z) \leq \min(V_A(x \rightarrow y), V_A(x))$ imply $V_A(z) \leq V_A(y)$.
- b. If A satisfy Definition 2.1(1) and condition (a), then, for any $\alpha, \beta \in [0, 1]$, $A_{(\alpha, \beta)}$ is a filter of L.

Proof:

- a. Assume that $A_{(\alpha, \beta)}$ is a filter of L for any $\alpha, \beta \in [0, 1]$. Since $V_A(z) \leq \min(V_A(x \rightarrow y), V_A(x))$, it follows that $V_A(z) \leq V_A(x \rightarrow y)$, $V_A(z) \leq V_A(x)$. Therefore, $x \rightarrow y \in A_{(t_A(z), 1-f_A(z))}$, $x \in A_{(t_A(z), 1-f_A(z))}$. As $V_A(z) \in [0, 1]$, and $A_{(t_A(z), 1-f_A(z))}$ is a filter of L, so $y \in A_{(t_A(z), 1-f_A(z))}$. Thus $V_A(z) \leq V_A(y)$.
- b. Assume A satisfy (a) and (b). For any $x, y \in L$, $\alpha, \beta \in [0, 1]$, we have $x \rightarrow y \in A_{(\alpha, \beta)}$, $x \in A_{(\alpha, \beta)}$, therefore $t_A(x \rightarrow y) \geq \alpha$, $1-f_A(x \rightarrow y) \geq \beta$ and $t_A(x) \geq \alpha$, $1-f_A(x) \geq \beta$, and so $\min(t_A(x \rightarrow y), t_A(x)) \geq \min(\alpha, \alpha) = \alpha$. By (a), we have $t_A(y) \geq \alpha$ and $1-f_A(y) \geq \beta$, that is, $y \in A_{(\alpha, \beta)}$. Since $V_A(I) \geq V_A(x)$ for any $x \in L$, it follows that $t_A(I) \geq \alpha$ and $1-f_A(I) \geq \beta$, that is, $I \in A_{(\alpha, \beta)}$. Then for any $\alpha, \beta \in [0, 1]$, $A_{(\alpha, \beta)}$ is a filter of L.

Theorem 2.14:

Let A be a vague filter of L, then for any $\alpha, \beta \in [0, 1]$, $A_{(\alpha, \beta)} (\neq \emptyset)$ is a filter of L.

Proof:

Since $A_{(\alpha, \beta)} \neq \emptyset$, there exist $\alpha, \beta \in [0, 1]$ such that $t_A(x) \geq \alpha$, $1-f_A(x) \geq \beta$. And A is a vague filter of L, we have $t_A(I) \geq t_A(x) \geq \alpha$, $1-f_A(I) \geq 1-f_A(x) \geq \beta$, therefore $I \in A_{(\alpha, \beta)}$. Let $x, y \in L$ and $x \in A_{(\alpha, \beta)}$, $x \rightarrow y \in A_{(\alpha, \beta)}$, therefore $t_A(x) \geq \alpha$, $1-f_A(x) \geq \beta$, $t_A(x \rightarrow y) \geq \alpha$, $1-f_A(x \rightarrow y) \geq \beta$. Since A is a vague filter of L, thus $t_A(y) \geq \min(t_A(x \rightarrow y), t_A(x)) \geq \alpha$ and $1-f_A(y) \geq \min(1-f_A(x \rightarrow y), 1-f_A(x)) \geq \beta$, it follows that $y \in A_{(\alpha, \beta)}$. Therefore, $A_{(\alpha, \beta)}$ is a filter of L.

Remark 2.15:

From Theorem 2.14, the filter $A_{(\alpha, \beta)}$ is also called a vague – cut filter of L.

Theorem 2.16:

Any filter F of L is a vague –cut filter of some vague filter of L.

Proof:

Consider the vague set A of L:

$A = \{(x, t_A(x)) / x \in L\}$, where If $x \in F$, $V_A(x) = \alpha$. If $x \notin F$, $V_A(x) = 0$. where $\alpha \in [0, 1]$. Since F is a filter of L, we have $1 \in F$. Therefore $V_A(I) = \alpha \geq V_A(x)$. For any $x, y \in L$, if $y \in F$, then $V_A(y) = \alpha = \min(\alpha, \alpha) \geq \min(V_A(x \rightarrow y), V_A(x))$. If $y \notin F$, then $x \notin F$ or $x \rightarrow y \notin F$. And so $V_A(y) = 0 = \min(0, 0) = \min(V_A(x \rightarrow y), V_A(x))$. Therefore A is a vague filter of L.

Theorem 2.17:

Let A be a vague filter of L. Then $F = \{x \in L / t_A(x) = t_A(I), 1-f_A(x) = 1-f_A(I)\}$ is a filter of L.

Proof:

Since $F = \{x \in L / t_A(x) = t_A(I), 1 - f_A(x) = 1 - f_A(I)\}$, obviously $I \in F$. Let $x \rightarrow y \in F, x \in F$, so $V_A(x \rightarrow y) = V_A(x) = V_A(I)$. Therefore $V_A(y) \geq \min(V_A(x \rightarrow y), V_A(x)) = V_A(I)$ and $V_A(I) \geq V_A(y)$, then $V_A(y) = V_A(I)$. Thus $y \in F$. It follows that F is a filter of L .

Case 1:($x = 1$). We have $V_C \wedge V_{A \cup B}(1) = V_C(1) \wedge V_A^{V_B} \approx V_B^{V_A}(1) = V_C(1) \wedge (V_A(1) \vee V_B(1)) = (V_C(1) \wedge (V_A(1) \vee V_B(1))) \vee (V_C(1) \wedge V_B(1)) = (V_C \wedge V_A)^{V_C \wedge V_B} \approx ((V_C \wedge V_B)^{V_C \wedge V_A}(1))$.

Case 2:($x \neq 1$).

$$\begin{aligned} & \bigvee_{\substack{p \neq 1, q \neq 1 \\ p * q \leq x}} \{V_C(x) \wedge V_A(p) \wedge V_C(x) \wedge V_B(q)\} \vee \\ & \{V_C(x) \wedge V_C(1) \wedge V_A^{V_B}(1) \wedge V_B(x)\} \vee \{V_C(x) \wedge \\ & V_C(1) \wedge V_B^{V_A}(1) \wedge V_A(x)\} = \bigvee_{\substack{p \neq 1, q \neq 1 \\ p * q \leq x}} \{V_C(x) \wedge \\ & V_A(p) \\ & \wedge V_C(x) \wedge V_B(q)\} \vee \{V_C(1) \wedge (V_A(1) \vee V_B(1))\} \\ & \wedge \{V_C(x) \wedge V_B(x)\} \vee \{V_C(x) \wedge V_A(x)\} = \end{aligned}$$

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$$\begin{aligned} & \bigvee_{\substack{p \neq 1, q \neq 1 \\ p * q \leq x}} \{V_C(x) \wedge V_A(p) \wedge V_C(x)\} \wedge V_B(q) \vee \{[\\ & (V_C(1) \wedge (V_A(1) \vee V_B(1))) \vee (V_C(1) \wedge V_B(1))] \wedge [\\ & V_C(x) \wedge V_B(x)] \vee (V_C(x) \wedge V_A(x))\} \leq \\ & \bigvee_{\substack{p \neq 1, q \neq 1 \\ p * q \leq x}} \{(V_C \wedge V_A)^{V_C \wedge V_B}(p \vee x) \wedge (V_C \wedge \\ & V_B)^{V_C \wedge V_A}(q \vee x)\} \vee [(V_C \wedge V_A)^{V_C \wedge V_B}(1) \wedge \\ & (V_C \wedge V_A)(p \vee x)] = [(V_C \wedge V_B)^{V_C \wedge V_A}(1) \wedge \\ & (V_C \wedge V_B)(q \vee x)] \\ & = \bigvee_{p * q \leq x} \{(V_C \wedge V_A)^{V_C \wedge V_B}(p \vee x) \\ & \wedge (V_C \wedge V_B)^{V_C \wedge V_A}(q \vee x)\}. \end{aligned}$$

CONCLUSION:

It holds that $V_C(x) \wedge$

In this paper, we introduced the concept of vague filters and we discuss some properties of Vague filters in terms of its level subsets. Also by introducing the notion of extended vague filters, it is proved that the set of all vague filters forms a bounded distributive lattice.

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