

Mathematical modelling and optimized flow characteristic study in a tube heat exchanger

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ABSTRACT:

The effect of heat flow and pressure drop on performance of tube heat exchanger was experimentally analysed and the process parameters were optimized. The process parameters were chosen for the experiment are velocity (0.5 to 1.0), spacing (0.5-0.7) and ellipticity (0.75-1.25). The outcomes of the experiment like heat transfer and pressure drop were measured at different conditions ordered in L9 orthogonal array. The main aim of this work is increase the heat transfer rate and reduce the pressure drop during functioning of tube heat exchanger. The ANOVA analysis was performed to identify the each parameter impact on outcomes. The experimental results reported that, at optimized condition (0.5-0.7-1.25) higher heat transfer, lower pressure drop at (1.0-0.7-1.25) was observed and reported. Finally single order regression equation was also developed for estimate the heat flow and pressure drop.

Keywords: Tube heat exchanger, heat flow, pressure drop, process parameters, ANOVA.

I. Introduction

The heat exchangers are commonly using devices for exchanging of heat in various process industries. The industrialists are continuously working on design and developing of better energy transfer and thermal treating systems since decades and they are known to be an excellent heat transfer systems. Mainly heat exchangers are applicable in condensation and evaporation sections of any refrigeration unit. The commercial applications of heat exchangers were found in power plants, radiators in automotive, chemical plant, waste heat recovery and pharmaceutical industries [1]. Most of the heat exchangers used in industries are tubular type heat exchanger such as shell and tube type heat exchanger, double pipe heat exchanger and plate heat exchanger, etc. [2]. The type of geometries used in plate and tube heat exchangers are circular or elliptical. The work reported on effect of rate of heat transfer through plate fin and circular tube heat exchanger having thin plate fins was discussed [3]. The other studies investigated on effect of heat flow rate in heat exchangers with circular and elliptical tubes was experimentally studied [4-8].

The experimentally studies reported that effect of external surface geometry [9], different geometrical parameters [10] and wavy geometries [11] on the performance of flat fin and circular tube heat exchangers was

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discussed. The similar works published on influence of louvered fin geometry on heat flow and pressure drop in flat tube heat exchanger was experimentally studied[12-15].

The experimental work study on effect of spacing among the fins of single row [16], number of tube rows, fin pitch and tube diameter [17] and equilateral staggered triangular and rectangular pitch [18] on heat transfer and pressure drop in plate and tube heat exchanger was deeply investigated. Similarly the experimental study investigate the influence of colburn and friction factors on heat flow and pressure drop in fin and tube heat exchangers was discussed [19-20].The work studied on investigate the effect of plate fin,fin structure on mass flow rate of flue gas in plate fin and tube heat exchanger was clearly analysed [21-23].The work reports on influence of inserts on heat transfer and pressure drop of tube and plate fin heat was experimentally studied [24].The progressive works reported on influence of thermal enhancement techniques on estimates the rise of Nusselt number of different input geometries was analyzed [25].

Further study reported on estimating the heat flow dynamic behaviour in the formation of wake and vortex shape at downstream of circular cylinder was addressed [26]. Another study reported on comparison of heat transfer conditions in the cross flow heat exchangers at different tube shapes was studied [27]. The work reported on experimental investigation on the effect of attack angle on heat and pressure drop of the cross flow heat exchanger having the staggered arrangement of flat tubes was studied [28]. Later the works reported on evaluate the characteristics of heat and pressure drop over the twisted tubes in cross flow heat exchanger was studied [29]. Another study investigated that fin thickness and rectangular winglet orientation on mass and thermal efficiency of the cross flow heat exchanger was discussed [30]. The new development in heat exchangers is that transfer of maximum heat at minimum pressure drop.The present work is focused on develop themathematical models and optimize the process parameters to get best condition for higher heat flow and lower pressure drop in tubular heat exchanger.

II. Experimental setup:

The experimental setup consists of a different parts were labeled in Fig 1. The test section consists of a U-bend double pipe heat exchanger; where inner tube is made of copper and annulus tube is made of cast iron. The inner tube is concentric to the annulus tube and fully enclosed by it. The hot fluid is pumped through the annular region and the water flows through the inner tube by using a pump. The mass flow rates for both the hot fluid and the water are controlled with by-pass valve arrangements. Throughout the experiments the mass flow rate of hot fluid through annulus is kept constant and the working fluid mass flow rate was observed at different flow conditions.

The process parameters used for conducting the experiment were tabulated in table 1.

Table 1: Levels of input parameters

Process parameters	Velocity (v)	Fin spacing (s)	Elipcticity (b/a)
1	0.5	0.5	0.75

2	0.75	0.6	1.0
3	1.0	0.7	1.25

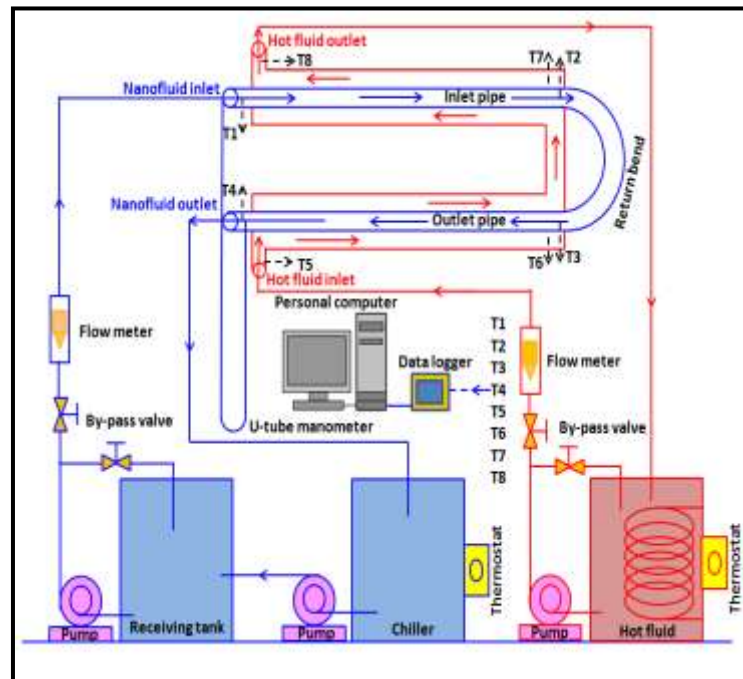


Fig 1: Schematic view of U-tube heat exchanger

III. Mathematical Analysis:

The process is assumed to be in steady state and the governing equations describing conservation of mass, momentum and energy are expressed as follows [31]:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad (1)$$

where ρ is the fluid density, t is the time and $\mathbf{U}$ is the flow velocity vector field.

Momentum equation:

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla (\rho \mathbf{U} \mathbf{U}) = -\nabla P + \nabla \tau + B \quad (2)$$

where P , τ and B are pressure, stress term, and the sum of the body forces, respectively.

Energy equation:

$$\frac{\partial (\rho h)}{\partial t} + \nabla (\rho \mathbf{U} C_p T) = \nabla \cdot (k \nabla T) \quad (3)$$

Where ρ is the density, h is the enthalpy and k is the effective thermal conductivity.

K- ϵ model:

The standard k-ε model [32] was used to describe the turbulence. The governing equations for the turbulent kinetic energy and turbulent kinetic energy dissipation are given as follows:

$$\frac{\partial(\rho k U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_j}{\partial x_i} - \rho \varepsilon \quad (4)$$

$$\frac{\partial}{\partial x_j} (\rho \varepsilon U_j) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] - \rho C_2 \frac{\varepsilon^2}{K + \sqrt{\partial \varepsilon}} \quad (5)$$

Where, σ_k and σ_ε are the turbulent Prandtl numbers for the turbulent kinetic energy and its dissipation.

Turbulent kinetic energy (k) and its dissipation rate (ε) are coupled to the $\left(\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \right)$ value as in the standard k-ε model. The empirical constants, C_2 , σ_k and σ_ε are equal to 1.9, 1.0 and 1.3 respectively [33]. The transport equation (5) was solved sequentially along with the discretized conservation equations. The discretization of the transport equations and conservation equations is conducted using the finite volume method [34] along with a second-order upwind scheme. The continuity equation and momentum equation are solved through an iterative scheme by using the coupling between pressure and velocity is employed through the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm [35]. The flue gas is assumed to have the properties of air. The cooling fluid circulating in the tube is water and it is considered to be fully developed turbulent flow. The convective heat transfer between the tube and fin is calculated using Gnielinski equation [36].

3.1 Development of mathematical model:

The responses of the experiment were measured in terms of Heat flow (HF) and Pressure drop (PD) and they defined as a function of velocity (V), Spacing (S) and ellipticity (b/a) as follows

$$HF = f(V, S, b/a) \quad (6)$$

$$PD = f(V, S, b/a) \quad (7)$$

The regression equation of any response term 'Y' with 'n' number of variables expressed as follows

$$Y = b_0 + \sum b_i X_i + \sum b_{ii} X_i^2 + \sum b_{ij} X_i X_j \quad (8)$$

Where b_0 = average of responses, b_i , b_{ii} = coefficient values and b_{ij} = interaction terms of the process parameters. The statistical software MINITAB was used to compute the coefficient terms of the regression equation and S/N ratio values. The optimum condition of response terms were analyzed with S/N ratio of larger the better and smaller the better as a quality character.

The S/N ratio for larger-the-better is given by

$$S/N = -10 \log_{10} \left(\frac{1}{n} \sum \frac{1}{y^2} \right) \quad (9)$$

Where n= number of repetitions, y= experimental response value.

Similarly S/N ratio for smaller the better is expresses as

$$S/N = -10 \log_{10} (y^2) \quad (10)$$

IV. Results and Discussion

The purpose of this study is to investigate the effects of three different parameters on heat flow and pressure drop in heat exchanger. The main parameter which effect the fluid flow and heat transfer are discussed in the present study. In the present study, input parameters were arranged in an L9 orthogonal array to find the optimum condition was shown in table 1.

4.1 Effect of input parameters on heat flow: Heat flow is the main response term, which define the quality characteristic of heat exchanger.

In order to assess the effect of process parameters on the heat flow the computed means and S/N ratio of each factor are calculated. In this study, the S/N ratio was chosen according to the criterion of “larger is better,” to maximize the heat flow. In Taguchi method, S/N ratio is used to find the deviation of the quality characteristics from desired value. S/N ratio is used to find the means of the response values corresponding parameter level shown in fig 2.

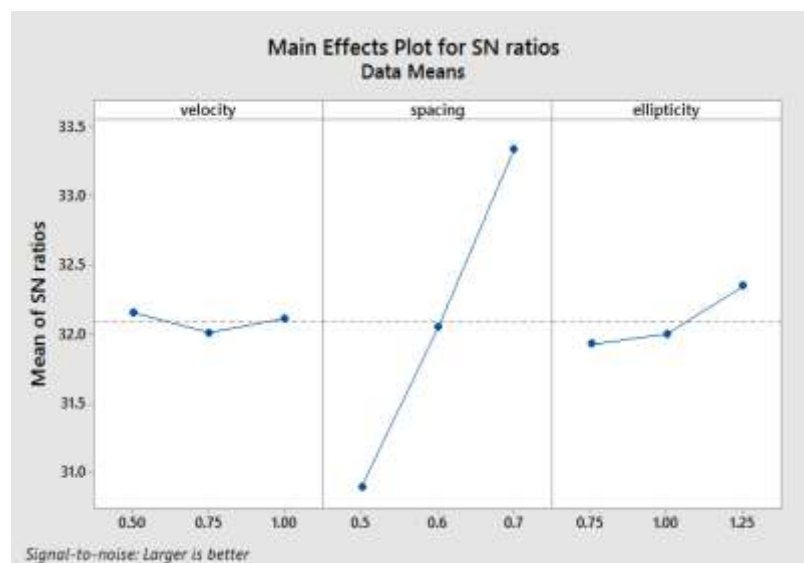


Fig 2: S/N ratio of heat transfer

From fig 2, it showed that at velocity of 0.5 m/sec higher S/N ratio was reported, similarly corresponding higher values of velocities as taken as 0.75 and 1 mm/sec are reported and plot the curve between velocities and means of S/N ratios. Spacing is the other factor which influence the heat transfer in heat exchanger. From the fig 2 it found that as the space values are increases corresponding S/N ratio values area also increased. S/N ratio was varies between 31 to 33.5 of spacing between 0.5 to 0.7. The third parameter ellipticity (b/a) influence on heat flow was also studied through S/N ratio values was plotted in Fig 2. The optimum condition to get the maximum heat flow from the heat exchanger was found from the fig 3 was reported as A1-B3-C3 as 0.5-0.7-1.25 was also shown in table 2.

4.2 Effect of input parameters on Pressure Drop (PD):

Pressure drop is the other measured term, which define the performance of heat exchanger. To evaluate the effect of input parameters on pressure drop was estimated from the computed means and S/N ratio values. In this study, the quality character S/N ratio of the term was chosen as “smaller is better” to minimize the pressure drop. S/N ratio is used to find the means of the response values corresponding parameter level was shown in fig 3.

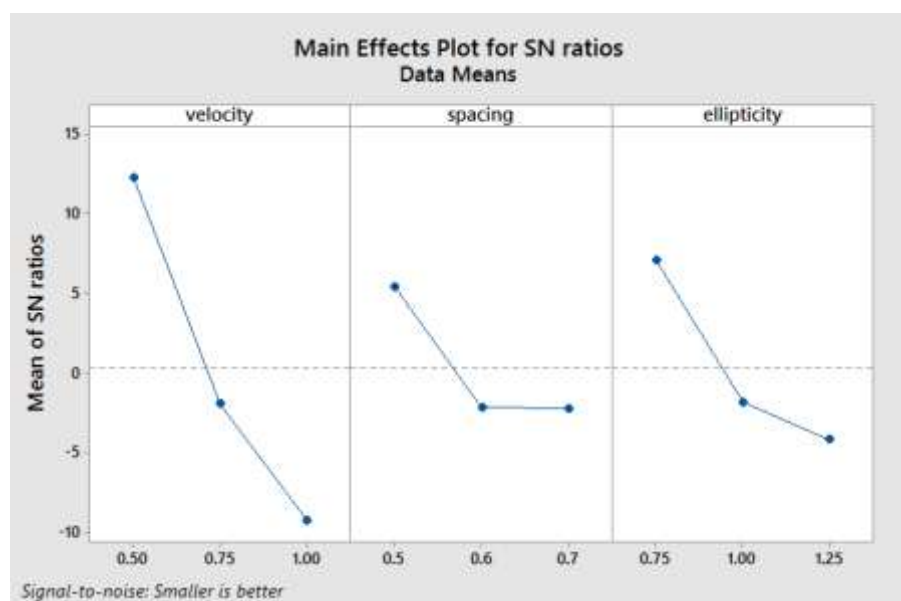


Fig 3: S/N ratio of Pressure drop

From fig 3, it was found that at lower velocity (0.5 m/sec) higher pressure drop was observed and it was drastically fall down where velocity term was increases from 0.75 to 1 m/sec. The other parameters spacing and ellipticity were also have the same effect on pressure drop as similar to the velocity as shown in fig 3. The optimum condition to get minimum pressure drop was found as A3-B3-C3 as 1.0-0.7-1.25, same was reported in Fig 3.

The input parameters and their corresponding observations are tabulated in table 2.

Table 2: Observations of the experiment

S No	Velocity	Spacing	Ellipticity	Heat Flow (Q) W	Pressure Drop (Pa)
1	0.5	0.5	0.75	33.96	0.029
2	0.5	0.6	1	41.04	0.725
3	0.5	0.7	1.25	47.72	0.702
4	0.75	0.5	1	34.02	1.137
5	0.75	0.6	1.25	40.05	1.301
6	0.75	0.7	0.75	46.4	1.349
7	1	0.5	1.25	37.2	4.726
8	1	0.6	0.75	39	2.26
9	1	0.7	1	45.17	2.329

4.3: Analysis of Residual plot:

A residual plot is a scatter plot that used to decide whether the experiment values are meet the assumptions of the model or not. In residual plot, computed data was fitted into different elements to check the adequacy of the developed mathematical model.

4.3.1: Heat Flow analysis:

The normal probability plot of residuals displays about the relation between residuals and their expected values where the normal distribution was happen. From Fig 4, it observed that residuals of the collected data was varies from -0.1 to 0.1 on placed x-axis and percentage value is 10 to 90 % placed on y-axis. The data points were placed very close and located both sides of the straight line shown in Fig.4, which means that experimented values of the heat flow were agreed with the assumptions of the mathematical model.

In the residuals versus fits graph, the fitted values 0.5 to 2 are keep on x-axis and residuals -0.10 to 0.10 on y-axis. The importance of this plot is where the data points were randomly distributed both sides of the 0 value. From the Fig 4, it observed that all data points were located both sides of the 0 value, so the measured heat flow values were meet the assumptions of the mathematical model. Similarly histogram is used to display the collected heat flow data graphically in the form of bars of different heights. The computed heat flow data was set into different frequency levels 0 to 3 were keep on y-axis shown in Fig 4. The computed residuals -0.10 to 0.15 of heat flow data was plotted on x-axis, which examine the collected data was distributed or skewed. From Fig 4, it was clear that different frequency level of bins were observed and data was skewed was reported. The order plot describes the order on which the data was collected, where the observation order taken on x-axis whose

scale of 1 to 9, whereas residual values -0.10 to 0.10 were keep on y-axis. From the interpolation of the data points it clear that the curve passes through all the experimented values of heat flow and occupied randomly around the centre line shown in Fig 4.

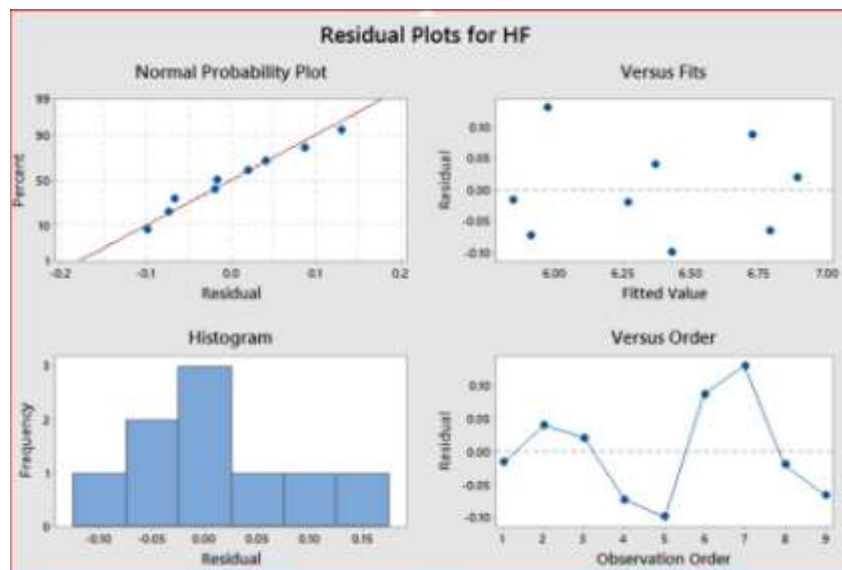


Fig 4: Residual plot of heat flow

4.3.2: Pressure drop analysis:

From Fig5 it clear that, the data collected from the observations was used to plot the graphwith residuals versus other elements. The normal probability plot displays about the relation between residuals and their expected values where the normal distribution was happen. Fig 5 showed that varied residuals -0.5 to +0.5 keep on x-axis and percentage is 10 to 90 % keep on y-axis. The plotted data points were very close and located both sides of the straight line shown in Fig.5, which means that experimented values of the pressure drop were agreed with the assumptions of the mathematical model. In the residuals versus fits graph, fitted values 6 to 7 are keep on x-axis and residuals -0.2 to +2 placed on y-axis. From the Fig 5, it observed that all the data points were located both sides of the 0 value, so the measured pressure drop values were meet the assumptions of the mathematical model. Likely histogram is used to graphically display the collected pressure drop data in the form of bars of different heights. The computed pressure drop data was set into different frequency levels 0 to 3 were keep on y-axis shown in Fig 5. The computed residuals -0.2 to + 0.3 of pressure drop data was plotted on x-axis and it reported that different frequency level of bins were observed and data was skewed was reported in Fig 5. Finally the order plot describes the order on which the data was collected, where the observation order taken on x-axis whose scale of 1 to 9, whereas residual values -0.2 to +0.2 were keep on y-axis. From the interpolation of the data points it clear that the curve passes through all the experimented values of pressure drop and occupied randomly around the centre line shown in Fig 5.

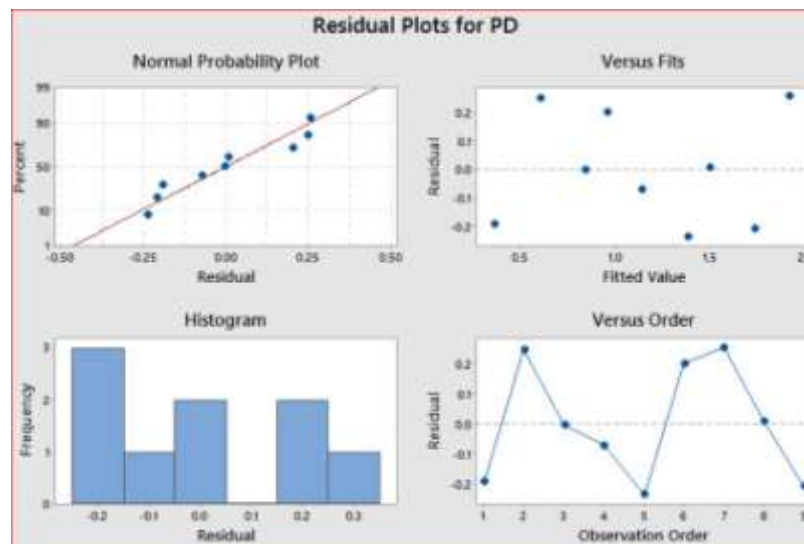


Fig 5: S/N ratio of pressure drop

4.4: ANOVA analysis:

The analysis of variance (ANOVA) was developed to establish the mathematical equation to predict the experiment outcomes.

The ANOVA test was performed to estimate the contribution of each parameter on response at 95 % of confidence level and P value is used to find the significant parameter which have less than 5% (0.05) value.

4.4.1: ANOVA analysis on heat flow:

The ANOVA analysis of heat flow describe about the significance of F and P value on heat flow. From the table 3 it clear that, R-sq value is more than 95% where the assumption of ANOVA test was agreed.

Table 3: ANOVA analysis of heat flow

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	199.46	66.48	43.29	0.001
Velocity	1	0.304	0.304	0.20	0.675
Spacing	1	199.91	193.91	126.27	0.000
Ellipticity	1	5.24	5.24	3.42	0.124
Error	5	7.67	1.53		

Total	8	207.14			
Model summary	S=1.23	R-sq=96.2%	R-sq (adj)=94.0 %	R-sq (Pred) =84.9 %	

The developed regression equation of heat flow in terms of single order variables was expressed as follows

$$HF = 3.33 - 0.90 (V) + 56.85 (S) + 3.47 (b/a) \quad (11)$$

4.4.2: ANOVA analysis on Pressure drop:

ANOVA analysis of pressure drop was discuss about the significant impact of P and F value on pressure drop. From the table 4, it was clear that the velocity term has got significant, because the P value of the velocity term is less than 5 % (0.005). The F value of velocity term was high as 29.78 more than other parameters. From the table 4, it clear that compared to other parameters the impact of velocity on pressure drop was high. The R-sq value is 87.33%, which was lesser than 95% of assumed value, but the velocity term got significant

Table 4: ANOVA analysis of pressure drop

Source	DF	Adj SS	Adj MS	F-Value	P-Value	
Regression	3	2.154	0.718	11.48	0.011	
Velocity	1	1.863	1.863	29.78	0.003	Significant
Spacing	1	0.002	0.002	0.04	0.859	
Ellipticity	1	0.289	0.289	4.62	0.084	
Error	5	0.312	0.062			
Total	8	2.467				
Model summary	S=0.25	R-sq= 87.33%	R-sq (adj)= 79.7%	R-sq (Pred) =51.5 %		

The formulated regression equation of pressure drop expressed in terms of process variables was represented as follows

$$PD = -2.86 + 5.24 (V) - 2.52 (S) + 2.06 (b/a) \quad (12)$$

V. Conclusions:

The following conclusions were drawn from the present study:

1. The impact of velocity, spacing and ellipticity on heat flow and pressure drop in tube heat exchanger was experimentally studied and analysed.
2. Better heat transfer and pressure drop values are found as the fin height is increased, due to the increased heat transfer surface area. The decrease in tube spacing causes the increase in heat transfer and decrease in pressure drop. As ellipticity rises in a tube, the heat transmitted across a heat changer increases.
- 3 The process parameters were optimized for get higher heat transfer at 0.5-0.7-1.25, and lower pressure drop at 1.0-0.7-1.25 was reported from S/N ratio.
4. ANOVA analysis was performed to estimate and identify the whether any parameter got significant or not, from the study it clear that velocity (0.003) term got significant and corresponding F and P values were also reported.
5. Regression equation was also developed to estimate and minimize the error percentage.

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