

Perception the Symbolic Scriptures Evaluation and Transformation of Elementary Algebra

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Abstract— *The current research work focus on two areas of unequal importance: Symbolic Expressions of Elementary Algebra ("ESA") on the one hand, and Problem-Based Discussion ("PBD") situations on the other. The main body of the paper exposes features Structural mottles, didactically plausible ESA systems. It is not a didactical study of algebra in the strict sense, which would be more or less directly applicable to what happens in classrooms, but rather a basic research on the structure of symbolic expressions that employ students and teachers in elementary algebra (ESA), intended to serve as a tool for subsequent didactic studies. "Understanding" ESAs means taking into account their (syntax, denotation, meaning and interpretation). According to the meaning we choose, the above statement may mean as well as the understanding of the ESAs is a collective, as the idea that the three components should not be separated. To tell the truth, we adhere to both propositions. However, the proposed study more specifically the didactic consequences of the second.*

Keywords— Elementary Algebra ESA, Problem-Based Discussion PDB, Symbolic Scriptures

1. Introduction

There is, at all, a very general hypothesis in this work, that of conceptual algebra [1]. The learner also faces linguistic difficulties linked to the complexity of the symbolic language of mathematics. In other words, the "transparency" of symbolic language is described as illusory, and one of the objectives of this work is precisely to lift this "illusion of transparency"[2]. The work originally chosen to study to begin the mistakes "Classics", as part of elementary algebra. To do this, it was developed a syntax of correct algebraic expressions, which was to serve as a reference for the study of errors. In doing so, developed a "step aside" to better address the didactical of algebra, in other words, put in brackets the teaching questions, to come back to them later. It was deliberately the party to reverse the usual perspective, compared to number of didactic studies, which is based on the teaching situation.

The research then followed its own development, determined by the constraints specific to the object of study. It was necessary to define precisely this object, in other words to determine the Expressions Symbols of Elementary Algebra (ESA) within the set of mathematical writings. It was hypothesized that ESAs could be described by a linguistic model (a grammar). Given the characteristics of these ESA, one model is better than another: If the mechanisms described to correspond to the representation of the "experts", this imposes such and such constraints ("plausibility"). If it is desired that the reader can appropriate the results of the research and adapt the model to his needs, this affects the presentation of research ("accessibility" and "adaptability") etc. It will not hide the fact that this modeling work, which was in principle a prerequisite for the study of errors, ended up interesting me for itself. This modeling of ESAs can be described as "fundamental" didactic research, or "pro-didactic" as it is located upstream of the didactics of algebra proper.

1.1. Objectives

Highlight, in an "accessible" way, the main characteristics of "structural models", didactically "plausible", "Systems" of Symbolic Writing in Elementary Algebra ". The difficulty is to allow the interlocutor to understand what it is and what the issues are, however, it was not in the difficult position of the physicist who

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 develops a highly abstract model of esoteric particles. My object is not exotic: they are less x and less y, sums and products, fractions and equations that students and their teachers handle every day. It is not the object which is difficult to explain, it is the method followed, the point of view adopted, and perhaps above all, the reasons to be interested in it. Symbolic writings of algebra elementary are indeed, for the teacher, common objects, "Transparent". What, then, may be the point of studying such trivial objects? Answering this question is to expose the choices that underlie this thesis. These choices have evolved with time. Min to put them in context, this evolution will be traced in this introductory chapter. For clarity of presentation, the statement of the purpose of the research will be regularly repeated (in box), highlighting the word or words that are more specifically explained in the following. I chose to include many elements about my personal approach. It is not about intellectual exhibitionism or self-indulgence, but as it will see in a way to make things clear by re-situating them in their context. This research therefore focuses on the symbolic writings of algebra Elementary

(ESA) type $a(b + c)$, $x+3=5$ or $\frac{x^2-1}{x^2+1} < 3$

The initial perspective of the research was quite different. The object of this first paragraph is to clarify this initial perspective, in order to inform the current choice to study ESAs.

Initially, the research was aimed at establishing a model unifying "classical" errors of elementary algebra. The perspective was then to "improve" the teaching of this part of mathematics, by taking positive account of these errors [2]. This assumes that errors are not the fortuitous product of inattention or ill will. but that they have their "reasons" (Krygowska 1987) [3] that the teacher must know. These "classic errors", also abundantly described in the literature.

2. Materials and methods

Consequently, what should have been a preliminary move has gradually become the focus of the research, namely the (wealthy and complicated) structure of the ESA scheme. Errors have gradually become an application, among others. At the same moment, the working technique and its presentation have developed: they are the topic of this paragraph. The post face will retrace the overall evolution of studies in methodology. With regard to the error model, it was developed the choice to place in the context of linguistics. This choice was based on two:

Hypothesis H1: The ESAs have a structure, and the linguistic tools are well adapted to describe this schwa.

It was not mean to say that algebra, or even mathematics, is a language. Such an utterance is of a philosophical order and goes beyond our purpose. A simply hypothesizing that an algebraic

expression such as $ax + b$ has a linguistic structure, just as the sentence the pet it cat is dead. What is common between the two is the fact of having a certain structure. It is obviously not the particular structure of the phrase or phrase '.

Hypothesis H2: The Chomsky's Genomic-trans informational grammar is an adequate formalism to describe the structure of the ESA.

The first hypothesis was initially the domain of the bet. The very existence of the theoretical construction which forms this research, i. e. the fact that ESAs are accurately described using a linguistic-type model, now validates this assumption.

Accessibility of the ESA model

Highlight, in an accessible way, the main Characteristics of Structural, Didactically Plausible Models, Systems of Symbolic Scripture in Elementary Algebra.

3. Algebraic Sums

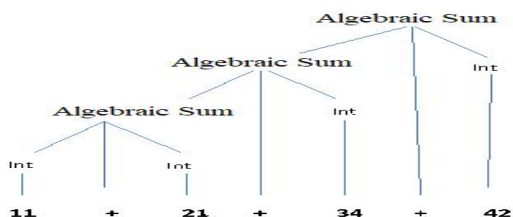
How to generate an algebraic sum of four terms such as:

$$11 + 21 + 34 + 42 ?$$

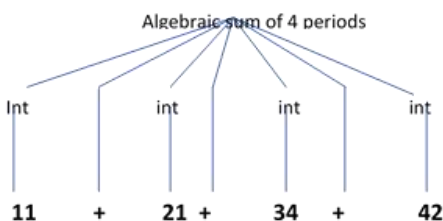
There are three solutions (at least). The first is to generate (in "profound form") three sums of two terms embedded in each other:

$$[[11 + 21] + 34] + 42$$

then remove (in "surface form") the supernumerary hooks. The second consists in directly saving (without going through a deep form) the algebraic sum by means of a so-called "recursive" rewriting rule, which gives it a structure of type



Finally, the third is to put in grammar an infinity different rewriting rules, one to generate the sums algebraic of three terms, one for those of four terms, one for those of five terms, etc. The structure of our example becomes:



3.1. Recursion

3.1.1. Recursive rewrite rules

Either to generate, by rewriting rules, an algebraic sum of type $a + b + c + d$. All that is needed is a recursive rewrite rule:

$$P := \left\{ \begin{array}{l} \emptyset \\ P \end{array} \right\} + \left\{ \begin{array}{l} a \\ b \\ c \\ d \end{array} \right\}$$

which reads:

"P is rewritten" continuation-empty "or P, followed by +, followed by a or b or neck d".

We can paraphrase it as follows:

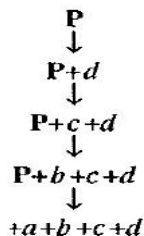
Any sequence of symbols in which the symbol P appears can be replaced by another sequence of symbols, which does not differ from the first only by the fact that each occurrence of P y is replaced:

- or by a single variable, chosen in $\{a b c d\}$,
- or by P himself, followed by "+" and a variable of $\{a b c d\}$ Rule:

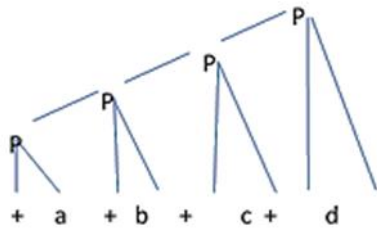
$$P := \left\{ \begin{array}{l} \emptyset \\ P \end{array} \right\} + \left\{ \begin{array}{l} a \\ b \\ c \\ d \end{array} \right\}$$

is called "recursive" because the categorical symbol P appears simultaneously on the left and on the right! Meta symbol "=" rewriting.

This same mechanism would make it possible to obtain: $a + b + b + bi - b$, $a + c + b + a$ or even $a + b$ alone. Here is the derivation (i.e the sequence of expressions obtained by successive applications of the rewrite rule) which results in $a + b + c + d$:



The phrase mark ("tree") associated with $a + b + c + d$ is the following



We will notice the similarity between this type of tree and that which Kirshner assigns to the deep form of $a + b + c + d$. The difference lies only in the absence of "parasitic" hooks

3.1.2. Recursive Generation of Algebraic Sum

Generating Algebraic Sums using recursive rules means that they will be rewritten a certain number (finite) (both in the form of a categorical symbol SA_n , (Algebraic Sum of n terms) followed by a chosen term in a suitable list, here is the corresponding recursive rewrite rule;

$$SA_n := \left\{ \begin{array}{l} \left(\begin{array}{l} \emptyset \\ + \\ - \\ \mp \end{array} \right) \left\{ \begin{array}{l} (Ide)^{SA_n} \\ (VAb) \\ Prn \\ Quo \\ Fra \\ Pui \\ Rac \\ Cli \\ Var \\ Vab \end{array} \right\} \left(\begin{array}{l} \emptyset \\ + \\ - \\ \mp \end{array} \right) \left\{ \begin{array}{l} (Ide) \\ (VAb) \\ Prn \\ Quo \\ Fra \\ Pui \\ Rac \\ Cli \\ Var \\ Vab \end{array} \right\} \end{array} \right.$$

We have framed the categorical symbol SA_n which appears in a paradigm on the right of the metasymbol of rewriting " := ", because it is he who makes the rule recursive.

It should be noted that the category SA_n is also included in the paradigms, among the category symbols placed in parentheses. In this way we can generate an algebraic sum formed of a variable and another sum put in parentheses, for example:

$$a + (b + c)$$

3.1.3. Evaluation

Like the type of nested structure:

$$[[a + b] + c] + d$$

that was in deep form, recursion to the left of the rules of rewriting sums and products favors their evaluation from left to right.

Let's first clarify what it means by "recursion on the left". "if A is a recursive element on the left, it dominates a tree that contains A somewhere on the leftmost branch". GLAOKu & MEL & JK (1972) speak of "development on the right" and thus comment on this expression: "Here we mean by direction of development, the direction in which the terminal symbols are" rejected "by example "C: = thread" is a development on the right. "The development on GLAOKU & MELEuK is therefore your recursion to the left of CII0MSKY, which does not clarify the discussion.

To evaluate recursively defined sums as just done it is enough for a recursive algorithm of the type:

To evaluate a sum:

evaluate the sum formed by all the terms save the last

evaluate the last term

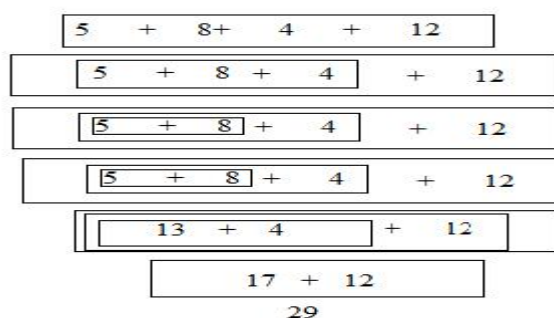
add

end of the evaluation

This algorithm is recursive because it recalls itself (at the step that we have emphasized). He will remember this way until the first term to be evaluated is a constant: Then be on the left of the expression. The leftmost sum will be evaluated, the result will be added to the term immediately to its right, and so on to the rightmost term. In other words, the evaluation will go from left to right.

It will illustrate this process by evaluate, and noting in numerical terms the numerical values of the constants (which are noted in italics).

$$So\ to\ evaluate\ 5 + 8 + 4 + 12$$



The evaluation algorithm does not differ from the one that operated on the deep form. As a result, the criticisms that applied to the square brackets for evaluation are still valid here. In particular, it is not because one evaluates a sum from left to right by default that it is necessary to prevent any other order of evaluation. Now, the meaning of recursion being fixed here (here: on the right), it becomes very difficult to conceive a simple rule if one wants to evaluate otherwise than from left to right. Indeed, to evaluate from right to left the expression:

$$25 + 1 + 99$$

It can be break it down into:

$$25 + 1 + 99$$

Sadly

$$1 + 99$$

does not have a sum structure, the only sums being here

$$25+1$$

and

$$25 + 1 + 99$$

(because the sums are generated by a recursive rule on the left). Therefore, an algorithm cannot be programmed to evaluate $1 + 99$ as a sum, as it does for the sum total (recursion). In addition, as part of the usual arithmetic at school, the evaluation of a sum such as:

$$11 + 21 + 34 + 42$$

is often done by means of a unique algorithm, that of the addition "in columns" which carries simultaneously on all the terms, without privileging any one of them. In this case, the decomposition from left to right in sums of two terms has no reason to be. It is therefore interesting to look for structural representations (and thus rewriting rules) that do not favor evaluation from left to right, or if it prefers that dissociate evaluation problems from questions of structure.

3.1.4. Transformation

We will see now that it is also difficult to transform in a simple way the structures generated recursively. However, this argument is based on the idea that the best characterization of transform sub expressions is "functional", which will be explained in advance, thus anticipating further research for the purposes of the discussion. To understand the following, it suffices to know that the notion of "syntactic function" covers that of "term" in the case an algebraic sum. Here is the definition of the syntactic function "s is a term of S" for a sum S_n (of more than two terms) defined recursively.

3.1.5. Term of a sum of two terms

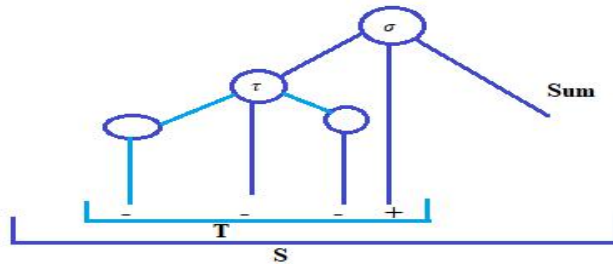
When there are only two terms, the definition of a term is as follows:

Let S be a terminal sequence and T a subset of S

Let c_i be a node of the phrase tree of S

Then, that T is a term de S, and write $T = t^s$, if there is a node τ of the syntagmatic tree of S such that:

- (i) T derives from τ
- (ii) τ is immediately dominated by σ
- (iii) σ is labeled Sum
- (iv) S derives from σ



3.1.6. Term of a sum of more than two terms

The definition of "term of a sum" is much more complicated when the sum has more than two terms.

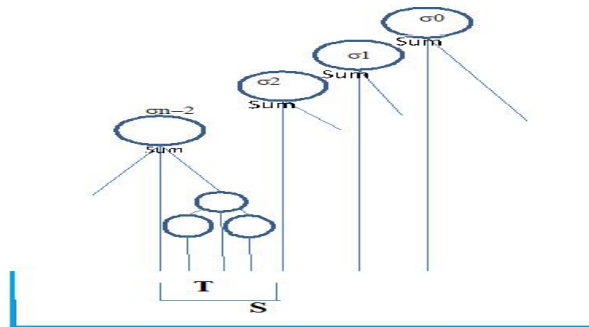
Let S be a terminal sequence and 'T' a subset of S

Let $(\sigma_0; \sigma_1; \dots; \sigma_{n-2})$ be a sequence of n-1 nodes of the tree

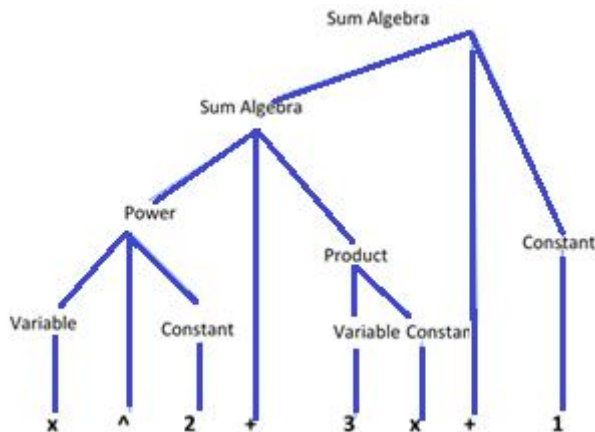
Syntagmatic of S

We say then that T is a term of S, and we write $T = t^s$, if it exists a node τ of the syntagmatic tree of S such that:

- (i) T derives from τ
- (ii) τ is not labeled Sum
- (iii) τ is immediately dominated by σ_{n-2}
- (iv) $\forall i \in [1; n - 2] \sigma_i$ is immediately dominated by σ_{i-1}
- (v) $\forall i \in [0; n - 2] \sigma_i$ is labeled sum
- (v) S derives from σ_0

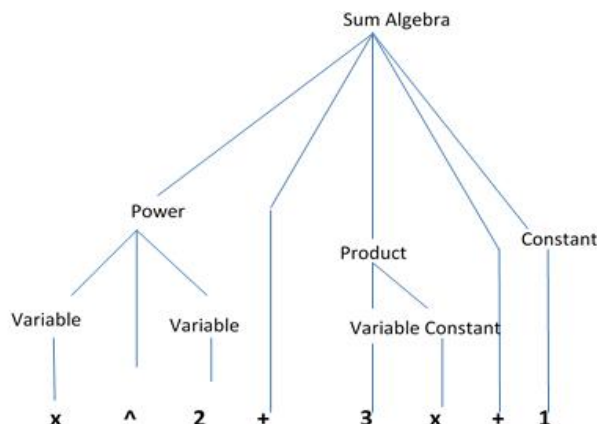


3.2. Example of a trinomial



In this example, see that $3x$ is a term of $x^2 + 3x + 1$ but not 3 (the rather complicated definition above is precisely made for this). The definition of "term of" is therefore more complex when a sum, defined recursively, comprises several terms. This complexity would be further increased if it defines the syntactic function "ith term of". In summary, recursively defined trees are rather dense. The main information they convey, namely that a sum of more than two terms is formed of a sum and a term is never used, except in the particular case of valuation from left to right. Finally, for most properties that concern expressions, such as transformations or syntactic functions, this information is an embarrassment that must be overcome. It is necessary to use mechanisms which are so many "principles" trees, which make inaccessible certain details, considered superfluous or even generative of complications (Mimer, 1984). All this leads us to give up recursive definitions. Our goal is to design a non-generative structure recursive, which provides the necessary information for

identifying and transforming the scriptures, but no more. This implies renouncing the category "Algebraic Sum" in general, and replacing it with new categories entitled "Algebraic Sum of 2 Terms", "Algebraic Sum of 3 Terms", ... "Algebraic Sum of 'N' Terms", etc. . Such that the different terms are then in the same ratio of syntactic function (in close order) with the sum (of i terms) as a whole.



3.3. Algebraic sum n terms

It was considered a priori that an algebraic sum is formed: either of two terms each preceded by a "+", "±" or more than two terms separated by a sign "+", "±" or "∓", the To lighten the scriptures note "T" the paradigm of the term of a sum:

$$T = \left\{ \begin{array}{l} (IDE) \\ (GVA) \\ Pro \\ quo \\ Pui \\ Rac \\ Cst \\ Cilvar \\ VAb \end{array} \right\}$$

Then the scheme of rewriting algebraic sums is:

$$SA_n := \left\{ \left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T \left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T \right\} \left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T \left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T \dots \right\}$$

Note that the notation:

$$\left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T$$

means that it is the term T preceded by its sign which is repeated, and not the T only, which would be the case if we had noted only:

$$\left\{ \begin{array}{l} + \\ - \\ \pm \end{array} \right\} T \dots$$

without putting this paradigm into braces.

4. Generalization: Expansion

All the above difficulties concern the exponentiation, and more precisely the exponent of powers. How is it possible? It was observed on the contrary that without the powers (and the roots), the ESA system, reduced to sums (binary), products (binary), differences and fractions would be elementary to describe, and part of this thesis would be almost irrelevant. We will briefly show two approaches, that of polynomial rings and a "categorical" point of view, which model ESAs very well. Without powers or roots in general.

It is perfectly obvious that the polynomial rings are a natural frame for expressions of the type:

$$a + 1$$

However, even when performing an extension in the domain of rational fractions with several variables, which makes it possible to

or:

$$x^2 + 2x + 2$$

To describe other types of extensions to the square roots, $\frac{a+1}{b+1}$ one is quickly limited when one wants to describe in all its generality the set of the algebraic expressions, which contains writings of the type:

$$\sqrt{\frac{3\sqrt{a} - 3\sqrt{b}}{\sqrt{a} - \sqrt{b}}}$$

Where "a" and "b" are variables,

In fact, the polynomial rings and their various extensions are only special cases of algebraic expressions, which as such are likely to be formally described by ad hoc grammars. This description seems to us one of the first works to be done as an extension of this paper, because it will test the adaptability of grammar, on a didactically interesting example. But even polynomials, even generalized ones, are only a very particular case of algebraic expressions and cannot account for the ESAs in their generality.

4.2. Didactic note

It should be noted that exponentiation, as an algebraic operation, is a sort of "blind spot" (or, at least, amblyope) of the teaching. Indeed, at the level of the college, this operation is defined classically (by generalizing the unary operations of elevation squared and cube), for an integer exponent, by repetition of the multiplication. It is only much later, in terminal, with the study of the exponential function, that exponentiation is introduced by a non-integer number ... and still not in all classes. A previous survey (Drouhard 1986) showed that only 10 out of 102 non-scientific students correctly answered an MCQ on the expression:

$$3\sqrt{x} \quad 4\sqrt{x}$$

Yet this type of know-how is considered a prerequisite of course by many teachers of higher education ... In fact, intrigued by this result, then stripped (quickly) 28 secondary textbooks, and realized that in the non-scientific series A and B two out of three collections did not discuss the notion of rational exponent at any time between the 3rd and the final included. The program of the literary final classes being, according to a well-established tradition, obtained by reduction of that of the scientific classes, it happens that this reduction bears precisely, among other things, in two manuals out of three, on the properties of the exponential function, and in particular about the concept of rational exponent.

It would be interesting, from a curricular perspective, to study how it is necessary to wait for the study of the exponential functions to approach the rational exponents, putting this delay in relation to the absence of the exponentiation of the classical structures ". From a conceptual point of view, the functions

$$x \rightarrow n\sqrt{x} \quad (n > 2)$$

are hardly more difficult to study than the function

$$x \rightarrow \sqrt{x}$$

which is discussed well before the terminal class. They are even more simple, when n is odd! As for the notation it is not harder to introduce than not to mention

$$x^{\frac{a}{b}}$$

it is not harder to introduce than

$$x^{-a}$$

not to mention

$$x^0$$

In other words, the theoretical framework of exponential functions, which happens "too late" for algebra. is also too powerful. It was not have need a theory of real exponent powers (and therefore, in particular, irrational) to define the rational exponent powers.

5. Conclusion

By studying well-formed expressions but of a character doubtful algebraic, it was shown that there can be no traction on the exponents of the powers and the orders of the roots, and consequently no restriction either on the graphemes to choose to form the variables. There is, however, another problem with the limitation of the ESA domain, that of understandable expressions whose grammaticality is difficult to determine. Indeed, in this domain as in that of natural languages, an expression can be understandable without being grammatical. This is the case of "gibberish": "it's understandable but it's not grammatical". In the "idiolect" [5] of the author, the product:

$$\frac{1}{2} x 3$$

is understandable but not acceptable, for lack of "x". This type of difficulty, specific to the products, will be discussed in the coming publications. In the meantime, going to study another question, that of sums and n-ary products. In any case, operate in this syntactical modeling work a reversal of perspective as to the definition of the ESAs, since it was not characterize the ESAs by the numbers they represent, it will define them henceforth as the writings engendered by our grammar, or if it prefer, the definition of the ESA strictly corresponds to the rewriting rules that engender them.

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