International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 07, 2020
ISSN: 1475-7192

ψ_1 - Operator Generated by Fuzzy Filter

Mohammed Majid Najm and Luay A. AL-Swidi

Abstract--- *In these paper we presented a new concept via fuzzy positional function properties were obtained and examples were provided to clarify these properties.*

Keywords--- *Fuzzy Set, Fuzzy Filter, Fuzzy Positional Function,* ψ_1*-Operator.*

I. INTRODUCTION

Zadeh is considered as the first researcher who established the notion of fuzzy set in 1965,Through his studies, has helped to remove ambiguity many Mathematical problems (1). Many scientists and researchers came after Zadeh and obtained surprising and surprising results. In addition, foggy groups had an important role in pure and applied mathematics and other disciplines such as computer science, electricity, mechanical, cryptography, and other sciences. Almohammed R. and Al-Swidi L.A introduced New Concepts of Fuzzy Local Function in 2019(2), also in same year introduced a New Types of Fuzzy Ψi- operator Generated by fuzzy ideal(3). Al-Razzaq AS and AL-Swidi LA In 2019 They classified the fuzzy sets theory as families [2], Well as in the same year they Finding and Taxonomy a New Fuzzy Soft points [3] and introduce the definition Soft Generalized Vague Sets [4].

II. PRELIMINARIES

In this section we will mention the most important concepts used in this paper. The pair $(\check{1}, \tau)$ is fuzzy topology space in Change [10]. $\check{\mathcal{F}}$ is collection fuzzy filter define in Lowen [5]. Where the triple $(\check{1}, \tau, \check{\mathcal{F}})$ is called a fuzzy filter topological space (in short F$\mathcal{FT}S$) .

Definition 2.1 [4].

A fuzzy set $\check{\mathcal{A}}$ is a set of ordered pairs consist of a generic element x and its membership's $f_A(x)$ define as. $\check{\mathcal{A}} = \{ (x, f_{\mathcal{A}}(x)), \forall\, x \in X, f_{\mathcal{A}} \in \mathcal{P}(\mathrm{X}, \mathrm{L})\}$.

Definition2.2.[4].

A fuzzy point denoted by P_y^λ for with support $y \in \mathrm{X}$ and $\lambda \in (0, 1]$ and the membership is $\mathrm{P}_y^\lambda(\mathrm{z}) = \begin{cases} \lambda \text{ if } z = y \\ 0 \text{ if } z \neq y \end{cases}$.

Definition 2.3.[11].

A fuzzy set $\check{\mathcal{A}}$ is called quasi-coincident with a fuzzy set $\breve{\mathcal{B}}$ denoted by $\check{\mathcal{A}}\mathrm{q}\breve{\mathcal{B}}$ iff $\exists\, z \in \mathrm{X} \ni f_{\mathcal{A}}(z) + g_{\mathcal{B}}(z) > 1$. Otherwise, $\check{\mathcal{A}}$ is not quasi-coincident with $\breve{\mathcal{B}}$ if $f_{\mathcal{A}}(w) + g_{\mathcal{B}}(w) \leq 1, \forall w \in \mathrm{X}$ and denoted by$\check{\mathcal{A}}\wp\breve{\mathcal{B}}$.

Definition 2.4.[9].

Let $\check{\mathcal{A}}, \breve{\mathcal{B}}$ any fuzzy set in Γ^{X}. The stander intersection, stander of union and complement are from.

Mohammed Majid Najm, Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Iraq. E-mail: mohammedmajid1995@yahoo.com
Luay A. AL-Swidi, Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Iraq. E-mail: drluayha11@yahoo.com

Received: 12 Feb 2020 | Revised: 15 March 2020| Accepted: 24 April 2020

1. $\breve{\mathcal{A}} \wedge \breve{\mathcal{B}}$ = {(x, min { $f_{\mathcal{A}}(x), g_{\mathcal{B}}(x)$}), $\forall x \in$ X }
2. $\breve{\mathcal{A}} \vee \breve{\mathcal{B}}$ = {(x, max { $f_{\mathcal{A}}(x), g_{\mathcal{B}}(x)$}), $\forall x \in$ X}
3. $1 - \breve{\mathcal{A}}$ = {(x, $1 - f_{\mathcal{A}}(x)$), $\forall x \in$ X }.

Definition 2.5. [9].

Let ($\breve{1}, \tau, \breve{\mathcal{F}}$) be a F$\mathcal{FTS}$. the fuzzy positional function of $\breve{\mathcal{A}}$ of the first type $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau)$ defined by:

$\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau) = \vee \{P_x^{\lambda};\ \forall\, \breve{u} \in\ q - \mathcal{N}(P_x^{\lambda}), \forall\, y\ \in\ X\ \text{s.t}\ max\{0, f_u(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for every $\breve{F} \in \breve{\mathcal{F}}\}$.
The fuzzy positional function of $\breve{\mathcal{A}}$ Denoted by $\breve{\mathcal{A}}^{\#1}$ or $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}})$ or $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}, \tau)$.

Definition 2.6 [10.11.12].

A subfamily m of Γ^X is called a fuzzy minimal structure if it satisfies the following:

1. $\breve{0}, \breve{1}$ belong to m.
2. If $\breve{\mathcal{A}}_i$ any fuzzy sets in m then $\vee_{i \in N} \breve{\mathcal{A}}_i \in m$. Where $i \in N$

The pair ($\breve{1}$, m) is called a fuzzy m- space.

III. A FUZZY ψ_1-OPERATOR

In this section we will introduce a new concept called A fuzzy ψ_1-operator while discussing the most important characteristics associated with this concept and giving examples showing those characteristics.

Definition 3.1.

Let ($\breve{1}, \tau, \breve{\mathcal{F}}$) be a F$\mathcal{FTS}$. A fuzzy ψ_1-operator: $\Gamma^X \longrightarrow \tau$ is defined as

$\psi_1(\breve{\mathcal{A}}) = \breve{1} - (\breve{1} - \breve{\mathcal{A}})^{\#1}$ for any $\breve{\mathcal{A}} \in \Gamma^X$.

$\psi_1(\breve{\mathcal{A}})$ is fuzzy open, since $(\breve{1} - \breve{\mathcal{A}})^{\#1}$ is fuzzy closed.

Example.3. 2.

Let ($\breve{1}, \tau, \breve{\mathcal{F}}$) be a F$\mathcal{FTS}$. Let X= {1, 2, 3}, A= {1, 2}, B= {1, 3}, C= {1, 2} and F= {1, 3}. The memberships of $\breve{\mathcal{A}}$, $\breve{\mathcal{B}}$, Č and F̌ are:

$$f_A(x) = \begin{cases} \frac{1}{2} \text{ if } x = 1 \\ \frac{7}{10} \text{ if } x = 2 \end{cases} \forall x \in \text{A},$$

$$g_B(x) = \frac{2x}{10}\ \forall x \in \text{B},$$

$$\text{K}_C(x) = \frac{\text{X}+7}{10}\ \forall x \in \text{C},$$

$$\text{h}_\text{F}(x) = \frac{3\text{x}+1}{10}\ \forall x \in \text{F}.$$

$\breve{\mathcal{A}}$={(1,0.5), (2, 0.7), (3,0)},

$\breve{\mathcal{B}}$={(1, 0.2), (2, 0), (3,0.6)},

$\breve{C}$={(1, 0.8), (2, 0.9), (3,0)},

$\breve{\mathcal{B}} \wedge \breve{C}$ = {(1, 0.2), (2, 0), (3,0)},

$\breve{\mathcal{B}} \vee \breve{C}$ = (1, 0.8), (2, 0.9), (3,0.6)},

$\breve{\mathcal{F}} = \{\breve{1}, \breve{F}\} \vee \{\breve{g}; \breve{g} \geq \breve{F}\}$.Where $\breve{F}$= {(1, 0.4), (2, 0), (3, 1)}.

Put $\tau = \{\breve{0}, \breve{1},, \breve{\mathcal{B}}, \breve{C}, \breve{\mathcal{B}}\wedge\breve{C}, \breve{\mathcal{B}}\vee\breve{C}\}$

$\psi_1(\breve{\mathcal{A}})$={ (1, 0.8), (2, 0.9), (3,0.6)}.

Theorem 3.3.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a F$\mathcal{FTS}$. $\breve{\mathcal{A}}, \breve{\mathcal{B}}$ be any two fuzzy sets in Γ^X then,

1. For each fuzzy set $\breve{\mathcal{A}}, \psi_1(\breve{\mathcal{A}})$ is open.
2. If $\breve{\mathcal{A}} \leq \breve{\mathcal{B}}$,then $\psi_1(\breve{\mathcal{A}}) \leq \psi_1(\breve{\mathcal{B}})$.
3. $\psi_1(\breve{\mathcal{A}}\vee\breve{\mathcal{B}})$= $\psi_1(\breve{\mathcal{A}})\vee\psi_1(\breve{\mathcal{B}})$.
4. $\psi_1(\breve{\mathcal{A}}\wedge\breve{\mathcal{B}}) = \psi_1(\breve{\mathcal{A}})\wedge\psi_1(\breve{\mathcal{B}})$.
5. $\psi_1(\breve{0}) = \breve{1} - \breve{1}^{\#1}$, and $\psi_1(\breve{1}) = \breve{1}$.
6. $\psi_1(\breve{\mathcal{A}}) \leq \psi_1\left(\psi_1(\breve{\mathcal{A}})\right)$
7. $\psi_1(\breve{\mathcal{A}}) = \psi_1\left(\psi_1(\breve{\mathcal{A}})\right) iff \left(\breve{\mathcal{A}}^c\right)^{\#1} = \left[\left(\breve{\mathcal{A}}^c\right)^{\#1}\right]^{\#1}$.
8. $\psi_1(\breve{\mathcal{A}} - F) = \psi_1(\breve{\mathcal{A}}) = \psi_1(\breve{\mathcal{A}}\vee F)$.for $F \notin \mathcal{F}$.
9. $\psi_{1_Y}(\breve{\mathcal{A}}) = \left[1 - \left(Y - \breve{\mathcal{A}}\right)^{\#1}\right\} \wedge Y$,for any $Y \subseteq X$.

Proof.

1. Since $\breve{\mathcal{A}}^{\#}$ is closed, thus $\psi_1(\breve{\mathcal{A}}) = [\left(\breve{\mathcal{A}}^c\right)^{\#1}]^{c}$ is open.
2. Let $\breve{\mathcal{A}} \leq \breve{\mathcal{B}}, So\ \breve{1} - \breve{\mathcal{B}} \leq \breve{1} - \breve{\mathcal{A}}$ by theorem.3.5 part (1) in [9], we have
$\left(\breve{1} - \breve{\mathcal{B}}\right)^{\#1} \leq \left(\breve{1} - \breve{\mathcal{A}}\right)^{\#1}$ *which imply that* $\breve{1} - \left(\breve{1} - \breve{\mathcal{A}}\right)^{\#1} \leq \breve{1} - \left(\breve{1} - \breve{\mathcal{B}}\right)^{\#1}$. Thus $\psi_1(\breve{\mathcal{A}}) \leq \psi_1(\breve{\mathcal{B}})$.
3. $\psi_1(\breve{\mathcal{A}}\vee\breve{\mathcal{B}}) = \left[\left(\left(\breve{\mathcal{A}}\vee\breve{\mathcal{B}}\right)^c\right)^{\#1}\right]^c = \left[\left(\breve{\mathcal{A}}^c\wedge\breve{\mathcal{B}}^c\right)^{\#1}\right]^c$
$= \left[\left(\breve{\mathcal{A}}^c\right)^{\#1}\wedge\left(\breve{\mathcal{B}}^c\right)^{\#1}\right]^c = [\left(\breve{\mathcal{A}}^c\right)^{\#1}]^{c}\vee[\left(\breve{\mathcal{B}}^c\right)^{\#1}]^{c} = \psi_1(\breve{\mathcal{A}})\vee\psi_1(\breve{\mathcal{B}})$.
4. $\psi_1(\breve{\mathcal{A}}\wedge\breve{\mathcal{B}}) = \left[\left(\left(\breve{\mathcal{A}}\wedge\breve{\mathcal{B}}\right)^c\right)^{\#1}\right]^c = \left[\left(\breve{\mathcal{A}}^c\vee\breve{\mathcal{B}}^c\right)^{\#1}\right]^c$
$= \left[\left(\breve{\mathcal{A}}^c\right)^{\#1}\vee\left(\breve{\mathcal{B}}^c\right)^{\#1}\right]^c = [\left(\breve{\mathcal{A}}^c\right)^{\#1}]^{c}\wedge[\left(\breve{\mathcal{B}}^c\right)^{\#1}]^{c} = \psi_1(\breve{\mathcal{A}})\wedge\psi_1(\breve{\mathcal{B}})$.
5. Clearly, $\psi_1(\breve{0}) = \breve{1} - \left(\breve{1} - \breve{0}\right)^{\#1} = \breve{1} - \left(\breve{1}\right)^{\#1}$.
Also, $\psi_1(\breve{1}) = \breve{1} - \left(\breve{1} - \breve{1}\right)^{\#1}$=$\breve{1} - \left(\breve{0}\right)^{\#1}$=$\breve{1} - \phi$=$\breve{1}$.
6. $\psi_1(\breve{\mathcal{A}}) = \breve{1} - \left(\breve{1} - \breve{\mathcal{A}}\right)^{\#1} \leq \breve{1} - \left(\left(\breve{1} - \breve{\mathcal{A}}\right)^{\#1}\right)^{\#1}$

$= \check{1} - \left(\check{1} - (\check{1} - \left(\check{1} - \check{\mathcal{A}}\right)^{\#1}\right)^{\#1} = \psi_1\left(\psi_1(\check{\mathcal{A}})\right)$

7. Since $\psi_1\left(\psi_1(\check{\mathcal{A}})\right) = \psi_1\left(\left[\left(\check{\mathcal{A}}^c\right)^{\#1}\right]^c\right)$

$= \left[\left([[[\check{\mathcal{A}}^c]^{\#1}]^c]^c\right)^{\#1}\right]^c = [[[\check{\mathcal{A}}^c]^{\#1}]^{\#1}]^c.$

So $\psi_1(\check{\mathcal{A}}) = \psi_1\left(\psi_1(\check{\mathcal{A}})\right) = \psi_1\left(\left[\left(\check{\mathcal{A}}^c\right)^{\#1}\right]^c\right) = \left[\left[\left(\check{\mathcal{A}}^c\right)^{\#1}\right]^{\#1}\right]^c$

$iff \left(\check{\mathcal{A}}^c\right)^{\#1} = \left[\left(\check{\mathcal{A}}^c\right)^{\#1}\right]^{\#1}.$

8. $\psi_1(\check{\mathcal{A}} - F) = (\check{\mathcal{A}} \wedge F^c)^{c^{\#1^c}} = [\check{\mathcal{A}}^c \vee F]^{\#1^c} \geq (\check{\mathcal{A}}^{c^{\#1}} \vee \phi)^c = \check{\mathcal{A}}^{c^{\#1^c}}$

. Thus $\psi_1(\check{\mathcal{A}}) \leq \psi_1(\check{\mathcal{A}} - F)$. But $\check{\mathcal{A}} - \check{F} \leq \check{\mathcal{A}}, then$

$\psi_1(\check{\mathcal{A}} - F) \leq \psi_1(\check{\mathcal{A}})$. also $\psi_1(\check{\mathcal{A}}) = \psi_1(\check{\mathcal{A}} - F)\ \forall F \notin \mathcal{F}$.

9. $\psi_{1_Y}(\check{\mathcal{A}}) = Y - [\left(Y - \check{\mathcal{A}}\right)^{\#1}{}_Y] = [\check{1} - [[\left(Y - \check{\mathcal{A}}\right)^{\#1}] \wedge Y] \wedge Y = [(\check{1} - \left(Y - \check{\mathcal{A}}\right)^{\#1}) \vee (\check{1} - Y)] \wedge Y = (\check{1} - \left(Y - \check{\mathcal{A}}\right)^{\#1} \wedge Y.$

Note 3.4.

If $\check{\mathcal{A}}, \check{\mathcal{B}}$ any two fuzzy sets in Γ^X then,

1. $\check{\mathcal{A}} \leq \check{\mathcal{B}}$, then $\psi_1(\check{\mathcal{B}}) \nleq \psi_1(\check{\mathcal{A}})$
2. $\psi_1(\psi_1(\check{\mathcal{A}})) \nleq \psi_1(\check{\mathcal{A}})$.

The following example shows that.

Let $(\check{1}, \tau, \check{\mathcal{F}})$ be a F$\mathcal{FTS}$. Let X = {1, 2, 3}= $\mathcal{A}$ {1,2,3}=$\mathcal{B}$= D, $\mathcal{H}$ = {3} and $\mathscr{b}$ = {1,2}.

The memberships of $\check{\mathcal{A}}, \check{\mathcal{B}}, \check{\mathcal{D}}, \check{\mathcal{H}}$ and $\check{\mathscr{b}}$ are:

$f_{\mathcal{A}}(x) = \frac{x+2}{10}\ \forall x \in \mathcal{A},\ g_{\mathcal{B}}(x) = \frac{x+5}{10}\ \forall x \in \mathcal{B},$

$K_{\mathcal{D}}(x) = \frac{10-x}{10}\ \forall x \in \mathcal{D}, L_{\mathcal{H}}(x) = \frac{3x}{10}\ \forall x \in \mathcal{H},$

$h_{\mathscr{b}}(x) = \frac{x+1}{10}\ \forall x \in \mathscr{b},$

$\check{\mathcal{A}}$={(1,0.3), (2,0. 4), (3,0.5)},

$\check{\mathcal{B}}$={(1,0.6), (2, 0.7), (3,0.8)},

$\check{\mathcal{D}}$={(1,0.9), (2, 0.8), (3,0.7)},

$\check{\mathcal{H}}$={(1,0), (2, 0), (3,0.9)},

$\check{\mathcal{D}} \wedge \check{\mathcal{H}}$ = {(1, 0), (2, 0), (3,0.9)}.

$\check{\mathcal{D}} \vee \check{\mathcal{H}}$={(1, 0.9), (2, 0.8), (3,0.9)}.

$\check{\mathcal{F}} = \{\check{1}, \check{\mathscr{b}}\} \vee \{\check{\mathcal{G}}; \check{\mathcal{G}} \geq \check{\mathscr{b}}\}$.Where $\check{\mathscr{b}}$= {(1, 0.2), (2, 0.3), (3, 0)}.

Put $\tau = \{\breve{0}, \breve{1}, \breve{\mathcal{D}}, \breve{\mathcal{H}}, \breve{\mathcal{D}} \wedge \breve{\mathcal{H}}, \breve{\mathcal{D}} \vee \breve{\mathcal{H}}\}$.

$\psi_1(\breve{\mathcal{A}})$= {(1, 0), (2, 0), (3,0.9)},

$\psi_1(\breve{\mathcal{B}})$={(1, 09), (2, 0.8), (3,0.9)},

Hence $\psi_1(\breve{\mathcal{B}}) \nleq \psi_1(\breve{\mathcal{A}})$

Also, $\psi_1(\psi_1(\breve{\mathcal{A}})) = \breve{1}$,

Thus $\psi_1(\psi_1(\breve{\mathcal{A}})) \nleq \psi_1(\breve{\mathcal{A}})$

Theorem 3.5.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a F$\mathcal{FTS}$. Let $\breve{\mathcal{F}}_1, \breve{\mathcal{F}}_2$ two fuzzy filters and $\breve{\mathcal{A}}$ be a fuzzy set, the following statement are hold:

1. If $\breve{\mathcal{F}}_1 \subseteq \breve{\mathcal{F}}_2$, then $\psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_1) \leq \psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_2)$.
2. $\psi_1(\breve{\mathcal{A}})(\breve{F}_1 \wedge \breve{F}_2)$=$\psi_1(\breve{\mathcal{A}})(\breve{F}_1) \wedge \psi_1(\breve{\mathcal{A}})(\breve{F}_2)$. where $\breve{F}_1 \breve{\in} \breve{\mathcal{F}}_1$ and $\breve{F}_2 \breve{\in} \breve{\mathcal{F}}_2$.

Proof.

1. $\breve{\mathcal{F}}_1 \subseteq \breve{\mathcal{F}}_2$ by Theorem 3. 8 part (1) in [9], we get $\breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_1) \leq \breve{\mathcal{A}}^{\#1}(\breve{\mathcal{F}}_2)$. Also is true when $(\breve{1} - \breve{\mathcal{A}})^{\#1}(\breve{\mathcal{F}}_1) \leq (\breve{1} - \breve{\mathcal{A}})^{\#1}(\breve{\mathcal{F}}_2)$ taking complement, we get $\psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_2) \leq \psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_1)$.

2. $\psi_1(\breve{\mathcal{A}})(\breve{F}_1) \wedge \psi_1(\breve{\mathcal{A}})(\breve{F}_2)$

$=[(\breve{1} - (\breve{1} - \breve{\mathcal{A}})^{\#1})(\breve{F}_1)] \wedge [((\breve{1} - (\breve{1} - \breve{\mathcal{A}})^{\#1})(\breve{F}_2)]$

$= \breve{1} - [((\breve{1} - \breve{\mathcal{A}})^{\#1})(\breve{F}_1) \vee (((\breve{1} - \breve{\mathcal{A}})^{\#1})(\breve{F}_2)]$ by theorem 3.8 part (2) in [9]

$=\breve{1} - [(\breve{1} - \breve{\mathcal{A}})^{\#1}(\breve{F}_1 \wedge \breve{F}_2)] = \psi_1(\breve{\mathcal{A}})(\breve{F}_1 \wedge \breve{F}_2)$.

Note 3.6.

If $\breve{A}$ any fuzzy set in Γ^X then, If $\breve{\mathcal{F}}_1 \subseteq \breve{\mathcal{F}}_2$ then $\psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_1) \nleq \psi_1(\breve{\mathcal{A}})(\breve{\mathcal{F}}_2)$.

The following Examples illustrate that.

Let $(\breve{1}, \tau, \mathcal{F})$ be a F$\mathcal{FTS}$, and X= {1, 2, 3} $=A = B$, $F_1 = \{1,2,3\}$,$F_2 = \{1, 3\}$,C={2}.The membership of $\breve{A}, \breve{B}, \breve{F}_1$, $\breve{F}_2$ and $\breve{C}$,are.

$f_A(x) = \frac{x+6}{10}\ \forall x \in$ A, $g_B(x) = \begin{cases} \frac{9}{10}\ if x\ is\ odd \\ \frac{8}{10} if x\ is\ even \end{cases}\ \forall x \in$ B, $K_C(x) = \frac{4x+1}{10}\ \forall x \in$ C, $h_{F_1}(x) = \frac{x+3}{10}\ \forall x \in F_2$, $h_{F_2}(x) = \frac{x}{10}\ \forall x \in F_1$.

$\breve{A}$={(1,0.7), (2, 0.8), (3,0.9)},

$\breve{1} - \breve{A}$={(1,0.1), (2, 0.2), (3,0.3)},

$\breve{B}$={(1, 0.9), (2, 0.8), (3,0.9)},

$\check{C}$={(1, 0), (2, 0.9), (3,0)},

$\check{B}\wedge\check{C}$= {(1, 0), (2, 0.7), (3,0)},

$\check{B}\vee\check{C}$={(1, 0.6), (2, 0.9), (3,0.8)}.

Let $\tau = \{\check{0}, \check{1}, \check{B}, \check{C}, \check{B}\wedge\check{C}, \check{B}\vee\check{C}\}$ and

$\check{\mathcal{F}}_1 = \{\check{1}, \check{F}_1\} \cup \{\check{\mathscr{g}}; \check{\mathscr{g}} \geq \check{F}_1\}$.Where $\check{F}_1$= {(1, 0.4), (2, 0.5), (3, 0.6)}.

$\check{\mathcal{F}}_2 = \{\check{1}, \check{F}_2\} \cup \{\check{\ell}; \check{\ell} \geq \check{F}_2\}$. Where $\check{F}_2$= {(1, 0.1), (2, 0), (3, 0)}. Then,

$\psi_1(\check{\mathcal{A}})(\check{\mathcal{F}}_1) = \check{1}$ And,

$\psi_1(\check{\mathcal{A}})(\check{\mathcal{F}}_2)$= {(1, 0.6), (2, 0.9), (3,0.4)}.

$\psi_1(\check{\mathcal{A}})(\check{\mathcal{F}}_1) \nleq \psi_1(\check{\mathcal{A}})(\check{\mathcal{F}}_2)$.

Theorem 3.7.

Let τ_1 and τ_2 be two fuzzy topologies such that τ_2 finer than τ_1 and $\check{\mathcal{A}}$ be any fuzzy set. For any fuzzy filter $\check{\mathcal{F}}$, then $\check{\mathcal{A}}^{\#1}(\tau_2, \check{\mathcal{F}}) \leq \check{\mathcal{A}}^{\#1}(\tau_1, \check{\mathcal{F}})$.

Proof.

Let $P_x^\lambda \,\tilde{\in}\, \check{\mathcal{A}}^{\#1}(\tau_2, \check{\mathcal{F}}), \forall\, \check{u} \in \tau_2$ s.t $q - \mathcal{N}(P_x^\lambda), \forall\, y \in X$ such that

$\max\{0, f_u(y) + g_{\mathcal{A}}(y) - 1\} \geq h_F(y)$ for some $\check{F} \in \check{\mathcal{F}}\}$, also is true for all $\check{\mathcal{V}} \in q\_\mathcal{N}(P_x^\lambda)$ in τ_1 because $\tau_1 \subseteq \tau_2$, we have that $P_x^\lambda \,\tilde{\in}\, \check{\mathcal{A}}^{\#1}(\tau_1, \check{\mathcal{F}})$.

Theorem 3. 8.

If τ_1, τ_2 are fuzzy topologies, τ_2 finer than τ_1. Then, $\psi_1(\check{\mathcal{A}})(\tau_1, \check{\mathcal{F}}) \leq \psi_1(\check{\mathcal{A}})(\tau_2, \check{\mathcal{F}})$.

Proof.

Let $(\check{1} - \check{\mathcal{A}}) \in \Gamma^X$, then

$(\check{1} - \check{\mathcal{A}})^{\#1}(\tau_2) \leq (\check{1} - \check{\mathcal{A}})^{\#1}(\tau_1)$ by Theorem 3.7. taking complement then

$(\check{1} - (\check{1} - \check{\mathcal{A}})^{\#1})(\tau_1) \leq (\check{1} - (\check{1} - \check{\mathcal{A}})^{\#1})(\tau_2)$, thus

$\psi_1(\check{\mathcal{A}})(\tau_1, \check{\mathcal{F}}) \leq \psi_1(\check{\mathcal{A}})(\tau_2, \check{\mathcal{F}})$.

Note 3.9.

Let $(\check{1}, \tau, \check{\mathcal{F}})$ be a F$\mathcal{FT}$S .then the following statement are hold.

1. If $\check{\mathcal{U}} \in \tau$, then $\check{\mathcal{U}} \leq \psi_1(\check{\mathcal{U}})$.

2. For each fuzzy set $\check{\mathcal{A}} \in \Gamma^X$, $int\check{\mathcal{A}} \leq \psi_1(\check{\mathcal{A}}) = int(\psi_1(\check{\mathcal{A}}))$.

Proof.

1. Let $\breve{\mathcal{U}} \in \tau, since\ \psi_1(\breve{\mathcal{U}}) = \breve{\mathcal{U}}^{c^{\#1^c}}$ and $\breve{\mathcal{U}}^c\ is\ closed, So\ \breve{\mathcal{U}}^{c^{\#1}} \leq cl\breve{\mathcal{U}}^c = \breve{\mathcal{U}}^c$.

This imples $\breve{\mathcal{U}} \leq \breve{\mathcal{U}}^{c^{\#1^c}} = \psi_1(\breve{\mathcal{U}})$. Thus $\breve{\mathcal{U}} \leq \psi_1(\breve{\mathcal{U}})$.

2. Since $int\breve{\mathcal{A}}$ is open, then by part(1) $int\breve{\mathcal{A}} \leq \psi_1(int\breve{\mathcal{A}})$ …(*)

Also, $int\breve{\mathcal{A}} \subset \breve{\mathcal{A}}$, by Theorem3.1. part(2),we have $\psi_1(int\breve{\mathcal{A}}) \leq \psi_1(\breve{\mathcal{A}})$…(**)

From (*) and (**), $int\breve{\mathcal{A}} \leq \psi_1(\breve{\mathcal{A}})$. Again since $int\breve{\mathcal{A}}$ is open $\psi_1(\breve{\mathcal{A}})$ then

$\psi_1(\breve{\mathcal{A}}) = int(\psi_1(\breve{\mathcal{A}}))$.Hence $int\breve{\mathcal{A}} \leq \psi_1(\breve{\mathcal{A}}) = int(\psi_1(\breve{\mathcal{A}}))$.

Definition 3.10.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a F$\mathcal{FT}$S , a fuzzy set $\breve{\mathcal{A}}$ is called $\breve{\mathcal{F}}$- Fuzzy dense if $A^{\#1} = \breve{1}$.

Example 3.11.

Let $(\breve{1}, \tau,)$be a F$\mathcal{T}$S, and X= {1, 2}, A, E, F are subset of X s.t

A={1, 2} with the memberships of $\breve{A}$, $T_A(x) = \begin{cases} \frac{x+8}{10}, x \in A \\ 0, x \notin A \end{cases}$

E={1,2} with the memberships of $\breve{E}$, $\begin{cases} \frac{4x}{10}, x \in A \\ 0, x \notin A \end{cases}$

F= {1} with the memberships of $\breve{F}$, $h_F(x) = \begin{cases} \frac{2x}{10}, x \in F \\ 0, x \notin F \end{cases}$

we get,

$\breve{A}$={(2, 0.9), (3,1)},

$\breve{E}$={(1, 0.4), (2, 0.8)},

Let $\tau = \{\breve{0}, \breve{1}, \breve{E}\}$ and

$\breve{\mathcal{F}} = \{\breve{1}, \breve{F}\} \cup \{\breve{g} \in \Gamma^X; \breve{g} \geq \breve{F}\}$.Where $\breve{F}$= {(1, 0.2), (2, 0)}.

$\breve{\mathcal{A}}^{\#1}$ =1,then $\breve{\mathcal{A}}$ is called $\breve{\mathcal{F}}$-dense.

Note 3.12.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a F$\mathcal{FT}$S . Then the following statement are hold:

1. $\breve{\mathcal{A}}^c\ is\ \mathcal{F} - dense\ iff\ \psi_1(\breve{\mathcal{A}}). = \breve{0}$.
2. $\breve{\mathcal{A}}\ is\ \mathcal{F} - dense\ iff\ \psi_1(\breve{\mathcal{A}}^c) = \breve{0}$.

Proof:

1. Clearly $\psi_1(\breve{\mathcal{A}}) = \breve{\mathcal{A}}^{c^{\#1^c}} = \breve{0}\ iff\ \breve{\mathcal{A}}^{c^{\#1}} = \breve{1}\ iff \breve{\mathcal{A}}^c is\ \mathcal{F} - dense.$

2. $\psi_1(\breve{\mathcal{A}}^c) = \breve{\mathcal{A}}^{c^{c^{\#1^c}}} = \breve{0}\ iff\ \breve{\mathcal{A}}^{\#1} = \breve{1}\ iff\ \breve{\mathcal{A}}\ is\ \mathcal{F} - dense.$

Received: 12 Feb 2020 | Revised: 15 March 2020| Accepted: 24 April 2020

Note 3.13.

Let $(\breve{1}, \tau, \breve{\mathcal{F}})$ be a F$\mathcal{FTS}$. Then $\breve{1} - \{P_x^\lambda\}$ is $\mathcal{F} - dense$ for any $P_x^\lambda \in \Gamma^X$ iff $\psi_1(\{P_x^\lambda\}) = \breve{0}$.

Proof.

$\breve{1} - \{P_x^\lambda\}$is $\mathcal{F} - dense\ iff\ (\breve{1} - \{P_x^\lambda\})^{\#1} = \breve{1}\ iff\ (1 - \{P_x^\lambda\})^{\#1^c} = \breve{0}$ iff $\psi_1(\{P_x^\lambda\}) = \breve{0}$.

Definition 3. 14.

Let $\breve{\mathcal{A}}$ be any fuzzy sets of $\breve{1}$, we define the collections σ and σ_0 by.

(1) $\sigma^1 = \{ \breve{\mathcal{A}} : \breve{\mathcal{A}} \leq \psi_1(\breve{\mathcal{A}})\}$.

(2) $\sigma_0{}^1 = \{ \breve{\mathcal{A}} : \breve{\mathcal{A}} \leq \text{int}\,(cl\ \psi_1(\breve{\mathcal{A}}))\}$.

Proposition 3. 15.

The collections $\sigma^1$and $\sigma_0{}^1$ are m_x-space.

Proof:

1. (i) by theorem 3.1 part(5) we have (1) $\psi_1(\breve{0}) = \breve{1} - \breve{1}^{\#1}$ then $\breve{0} \leq \psi_1(\breve{0})$ thus $\breve{0} \in \sigma^1$. Also $\psi_1(\breve{1}) = \breve{1} - \breve{0}^{\#1} = \breve{1}$. then $\breve{1} \leq \psi_1(\breve{1})$ thus $\breve{1} \in \sigma^1$.

   (ii) let $\{\breve{\mathcal{A}}_\gimel : \gimel \in \Lambda\} \subseteq \sigma^1. So\ \breve{\mathcal{A}}_\gimel \leq \psi_1(\breve{\mathcal{A}}_\gimel)\ \forall \gimel \in \Lambda\ imply\ that\ \breve{\mathcal{A}}_\gimel \leq \psi_1(\breve{\mathcal{A}}_\gimel) \subseteq \psi_1(\vee \breve{\mathcal{A}}_\gimel)\ that\ is\ \vee \breve{\mathcal{A}}_\gimel \subseteq \psi_1(\vee \breve{\mathcal{A}}_\gimel)$, *thus* $\vee \breve{\mathcal{A}}_\gimel \in \sigma^1$.

2. (i) Let $\breve{\mathcal{A}} \in \sigma^1, then\ we\ have\ \breve{\mathcal{A}} \subseteq \psi_1(\breve{\mathcal{A}})$ *and since by Theorem* 3.1 part (1) $\psi_1(\breve{\mathcal{A}})$ is open that is int $(\psi_1(\breve{\mathcal{A}})) = \psi_1(\breve{\mathcal{A}})$. Thus $\breve{\mathcal{A}} \subseteq \psi_1(\breve{\mathcal{A}}) \subseteq \text{int}(cl\ \psi_1(\breve{\mathcal{A}}))$. Hence$\breve{\mathcal{A}} \in \sigma_0{}^1$.Thus $\sigma^1 \subseteq \sigma_0{}^1$. Since $\breve{0}, \breve{1} \in \sigma^1$, thus $\breve{0}, \breve{1} \in \sigma_0{}^1$.

   (ii) Let $\breve{\mathcal{A}}_\gimel \in \sigma_0{}^1\ \forall \gimel \in \Lambda\ then\ \breve{\mathcal{A}}_\gimel \leq \text{int}(cl\ \psi_1(\breve{\mathcal{A}})) \leq \text{int}(cl\ \psi_1(\vee \breve{\mathcal{A}}_\gimel))$ implies that $\vee \breve{\mathcal{A}}_\gimel \leq \vee\ \text{int}(cl\ \psi_1(\breve{\mathcal{A}})) \leq \vee\ \text{int}(cl\ \psi_1(\vee \breve{\mathcal{A}}_\gimel)) = \text{int}\,(cl\ \psi_1(\vee \breve{\mathcal{A}}_\gimel))$. Thus $\vee \breve{\mathcal{A}}_\gimel \in \sigma_0{}^1$.

REFERENCES

[1] Zadeh, Lotfi A. "Fuzzy sets." *Information and control* 8.3 (1965): 338-353.

[2] Almohammed, Raghad, and Luay A. AL-Swidi. "Generate a New Types of Fuzzy Ψ_i-Operator." *International Conference on Information, Communication and Computing Technology. Springer, Cham*, 2019.

[3] Almohammed, Raghad, and Luay A. AL-Swidi. "New Concepts of Fuzzy Local Function." *Baghdad Science Journal* 17.2 (2020): 515-522.

[4] Razzaq, Audia Sabri Abd Al, and Luay Abd Al Hani Al Swidi. "On Classification of Fuzzy Set Theory". *J. Modern. Math. Statistics.* (2019).

[5] Razzaq, Audia Sabri Abd Al, and Luay Abd Al Hani Al Swidi. "Finding and Taxonomy a New Fuzzy Soft Points". *J. Modern. Math. Statistics.* (2019).

[6] Razzaq, Audia Sabri Abd Al, and Luay Abd Al Hani Al Swidi. "Soft Generalized Vague Sets: An application in Medical Diagnosis." *Indian Journal of Public Health Research & Development* 10.2 (2019): 901-907.

[7] Chang, Chin-Liang. "Fuzzy topological spaces." *Journal of mathematical Analysis and Applications* 24.1 (1968): 182-190.

[8] Lowen, R. "Convergence in fuzzy topological spaces." *General Topology and its Applications* 10.2 (1979): 147-160.

[9] Nejm, Mohammed Majid, and Luay Abd Al Hani Al Swidi. ″Fuzzy Positional Function Via Fuzzy Filter″. *International Journal of Recent Technology and Engineering* (2020):511-515.

[10] Bhattacharyya, Anjana. "Several concepts of continuity in fuzzy m-space." *Annals of Fuzzy Mathematics and Informatics* 13.2 (2017): 5-21.

[11] Alimohammady, M., and M. Roohi. "Compactness in fuzzy minimal spaces." *Chaos, Solitons & Fractals* 28.4 (2006): 906-912.

[12] Alimohammady, M., and M. Roohi. "Fuzzy minimal structure and fuzzy minimal vector spaces." *Chaos, Solitons & Fractals* 27.3 (2006): 599-605.