International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 08, 2020
ISSN: 1475-7192

Stochasticity of two preys and one predator environmental framework utilizing Fourier tool

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ABSTRACT--*This article manages an investigation on numerical model of an ecological system with two prey, one predator and also within the sight of arbitrarily fluctuating main impetuses on the development of the species at time 't' of a customary eco framework. The model is depicted by a couple of non-direct differential conditions. Stochastic steadiness, as far as the fluctuations of the populaces of the given framework is inferred by using Fourier transform tool.*

Key words--S*tability, Fourier transform, equilibrium point, stochastic stability, variance*

I. INTRODUCTION

Demonstrating the changing parts of a biology framework is the best beneficial way to deal with capture the complexity of characteristic environmental factors i.e examining the collaborations among species and development of the species populace. The prey-predator standards were first exhibited by Lotka-Volterra [1-5]. They utilized basic reaction work corresponding to the quantity of predators. In prey-predator models, species typically follow distinctive development capacities and among these [6-8], Logistic development work is significant one, which was first utilized by Verhulst [2] for humanoid development. Moreover Feller [9] assumed that pretty much every mass that builds asymptotically will reasonable to the Logistic development rule to some augmentation. There likewise exist some extra development capacities proposed by Gompertz [10], May [11]. There are a few sorts of reaction capacities, for example, proportion subordinate, Holling types reaction capacities [12], Michaelis Menten type, Beddington-DeAngelis [13,14] reaction work, and so on. Numerical models of natural plans, mirroring these stresses, have been prosecuted to look at the unfaltering quality of a decent variety of frameworks. For instance, see [15-24]. The vivacious affiliation concerning executioner and their casualty has for quite some time been and will suffer to be one of the main topics in environment because of its broad criticalness [17]. Different extraordinary works have been accomplished for the Lotka–Volterra type predator–prey plot yet significant uncertainties and errors in explanation incited out this training. In ref. [18], Holling suggested that there exist three useful reaction of the predator which generally called Holling type I, Holling type II and Holling type III [25]. Samantha et al [23] proposed a scientific model for a prey predator model and they examined the steadiness about inside balance point. Due to this inspiration, we built the accompanying stochastic model and furthermore examined the soundness at inside consistent state alongside some numerical reproductions.

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Received: 21 Dec 2019 | Revised: 18 Jan 2020 | Accepted: 05 Feb 2020

II. STOCHASTIC ANALYSIS

This segment is intended for the augmentation of the deterministic model of [23], which is framed by including loud term. There are a few manners by which natural clamour might be fused in an environmental framework. Outer commotion may emerge from irregular changes of limited number of parameters around some known mean estimations of the populace densities. Since the oceanic biological system which consistently has unsystematic variances of the earth, it is hard to characterize the standard marvel as a deterministic perfect. The stochastic examination benefits us to get an additional instinct about the constant changing parts of any environmental unit. The two prey species develop strategically and direct challenge is considered between them. A pictorial representation of this biological phenomenon is as below in figure (2.1).

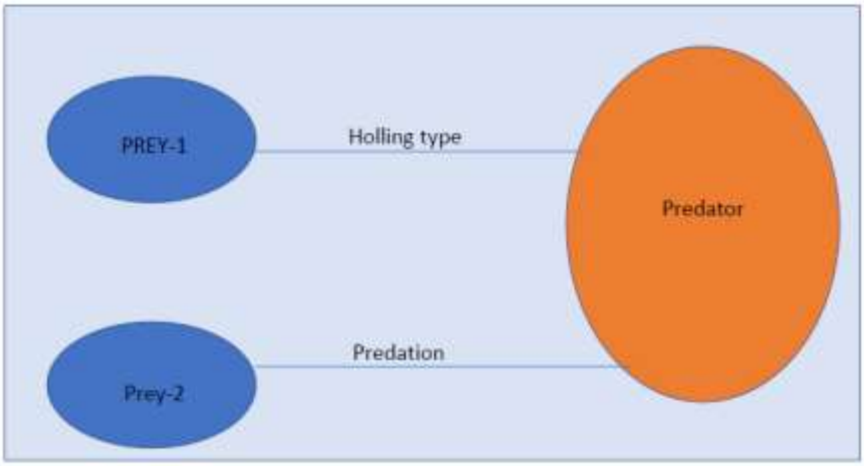

Figure-(2.1)

The deterministic model given by [23] is stretched out with the impact of irregular commotion of the ecological outcomes in a stochastic framework given beneath.

$$x'(t) = r_1 x\left(1 - \frac{x}{K}\right) - \frac{cxz}{a + \alpha\eta y + x} + \alpha_1 \xi_1(t) \tag{2.1}$$

$$y'(t) = \beta y - \delta yz + \alpha_2 \xi_2(t) \tag{2.2}$$

$$z'(t) = \frac{bxz}{a + \alpha\eta y + x} + \gamma yz - mz + \alpha_3 \xi_3(t) \tag{2.3}$$

where $x(t)$, $y(t)$ and $z(t)$ indicate the populace thickness of first prey, second prey and predator individually. We expect that the development of the subsequent prey is exponential, so there is a colossal stock of the second prey in the populace without predator. In this manner, there is no scanning time of the predator for the subsequent prey. Here $r_1, K, a, b, c, m, \alpha, \beta, \eta, \delta, \gamma$ are on the whole positive. r_1, K individually speak to the inborn development rate and conveying limit of the principal prey. speak to the inherent development rate and predation pace of the second prey separately. γ is the effectiveness with which the subsequent prey devoured by the predator gets changed over into predator biomass and is the passing pace of the predator without prey, $\alpha_1, \alpha_2, \alpha_3$ are the genuine constants and $\xi_i(t) = [\xi_1(t), \xi_2(t), \xi_3(t)]$ is a three dimensional Gaussian white noise process fulfilling $E(\xi_i(t))=0;\ i=1,\ 2,\ 3;\ E[\xi_i(t)\xi_j(t)]=\delta_{ij}\delta(t-t');\ i=j=1,\ 2,\ 3$ where δ_{ij} is the Kronecker delta function; δ is the Dirac delta function. Where δ_{ij} is the Kronecker sign; δ is the δ-dirac function. All other parameters have their own usual meanings [23]

Let $x(t)=u_1(t)+S^*;\ y(t)=u_2(t)+P^*;\ z(t)=u_3(t)+T^*;$ (2.4)

Then $x'(t)=u_1'(t);y'(t)=u_2'(t);z'(t)=u_3'(t)$ (2.5)

Utilizing (2.4) and (2.5), the linear parts of (2.1), (2.2) and (2.3) are

$$u_1'(t)=-\frac{r_1}{K}u_1(t)S^*-cu_3(t)S^*+\alpha_1\xi_1(t) \tag{2.6}$$

$$u_2'(t)=\alpha_2\xi_2(t) \tag{2.7}$$

$$u_3'(t)=bu_1(t)T^*+\gamma u_2(t)T^*+\alpha_3\xi_3(t) \tag{2.8}$$

Taking the Fourier transform on the two sides of (2.6), (2.7), (2.8) we get,

$$i\omega\tilde{u}_1(\omega)=-\left(\frac{r_1S^*}{K}\right)\tilde{u}_1(\omega)-cS^*\tilde{u}_3(\omega)+\alpha_1\xi_1(\omega) \tag{2.9}$$

$$i\omega\tilde{u}_2(\omega)=\alpha_2\xi_2(\omega) \tag{2.10}$$

$$i\omega\tilde{u}_3(\omega)=bT^*\tilde{u}_1(\omega)+\gamma T^*\tilde{u}_2(\omega)+\alpha_3\xi_3(\omega) \tag{2.11}$$

The matrix form of (2.9)-(2.11) is $M(\omega)\tilde{u}(\omega)=\tilde{\xi}(\omega)$ (2.12)

where $M(\omega)=\begin{pmatrix} i\omega+\frac{r_1S^*}{K} & 0 & cS^* \\ 0 & i\omega & 0 \\ -bT^* & -\gamma T^* & i\omega \end{pmatrix}$; $\tilde{u}(\omega)=\begin{bmatrix}\tilde{u}_1(\omega)\\ \tilde{u}_2(\omega)\\ \tilde{u}_3(\omega)\end{bmatrix}$; $\tilde{\xi}(\omega)=\begin{bmatrix}\alpha_1\tilde{\xi}_1(\omega)\\ \alpha_2\tilde{\xi}_2(\omega)\\ \alpha_3\tilde{\xi}_3(\omega)\end{bmatrix}$;

Equation (2.12) can also be written as $\tilde{u}(\omega)=\left[M(\omega)\right]^{-1}\tilde{\xi}(\omega)$ (2.13)

where $\left[M(\omega)\right]^{-1}=\frac{1}{R(\omega)+iI(\omega)}\begin{pmatrix} A_1 & D_1 & G_1\\ B_1 & E_1 & H_1\\ C_1 & F_1 & I_1\end{pmatrix}$ (2.14)

and where $A_1=-\omega^2;B_1=0;C_1=i\left(b\omega T^*\right);D_1=-c\gamma S^*T^*;E_1=\left(-\omega^2+bcS^*T^*\right)+i\left(\frac{r_1\omega S^*}{K}\right)$;

$F_1=\left(\frac{r_1\gamma T^*S^*}{K}\right)+i\left(\omega\gamma T^*\right);G_1=i\left(-\omega cS^*\right);H_1=0;I_1=\left(-\omega^2\right)+i\left(\frac{\omega r_1S^*}{K}\right)$.

Here $|A_1|^2=X_1^2+Y_1^2$; $|B_1|^2=X_2^2+Y_2^2;|C_1|^2=X_3^2+Y_3^2;|D_1|^2=X_4^2+Y_4^2;|E_1|^2=X_5^2+Y_5^2$;

$|F_1|^2=X_6^2+Y_6^2;|G_1|^2=X_7^2+Y_7^2;|H_1|^2=X_8^2+Y_8^2;\ |I_1|^2=X_9^2+Y_9^2$;

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where $X_1 = -\omega^2$; $Y_1 = 0$; $X_2 = 0$; $Y_2 = 0$; $X_3 = 0$; $Y_3 = b\omega T^*$; $X_4 = -c\gamma S^* T^*$; $Y_4 = 0$;

$$X_5 = \left(-\omega^2 + bcS^* T^*\right); Y_5 = \left(\frac{r_1 \omega S^*}{K}\right); X_6 = \left(\frac{r_1 \gamma T^* S^*}{K}\right); \; Y_6 = \left(\omega \gamma T^*\right); X_7 = 0; \; Y_7 = \left(-\omega c S^*\right);$$

$$X_8 = 0; \; Y_8 = 0; X_9 = \left(-\omega^2\right); Y_9 = \left(\frac{\omega r_1 S^*}{K}\right); \tag{2.15}$$

$$\left|M(\omega)\right|^2 = [R(\omega)]^2 + [I(\omega)]^2 \text{ where } R(\omega) = \left(-\frac{\omega^2 r_1 S^*}{K}\right) \text{ and } I(\omega) = cbS^* T^* \omega - \omega^3$$

If the function $Y(t)$ has a zero mean value, then the fluctuation intensity (variance) of its components in the frequency interval $[\omega, \omega + d\omega]$ is $S_Y(\omega)d\omega$. where $S_Y(\omega)$ is spectral density of Y and is defined as

$$S_Y(\omega) = \lim_{\tilde{T} \to \infty} \frac{\left|\tilde{Y}(\omega)\right|^2}{\tilde{T}} \tag{2.16}$$

If Y has a zero mean value, the inverse transform of $S_Y(\omega)$ is the auto covariance function

$$C_Y(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_Y(\omega) e^{i\omega\tau} d\omega \tag{2.17}$$

The corresponding variance of fluctuations in $Y(t)$ is given by

$$\sigma_Y{}^2 = C_Y(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_Y(\omega) d\omega \tag{2.18}$$

and the auto correlation function is the normalized auto covariance

$$P_Y(\tau) = \frac{C_Y(\tau)}{C_Y(0)} \tag{2.19}$$

For a Gaussian white noise process, it is

$$S_{\xi_i \xi_j}(\omega) = \lim_{\tilde{T} \to +\infty} \frac{E\left[\tilde{\xi}_i(\omega) \tilde{\xi}_j(\omega)\right]}{\tilde{T}} = \lim_{\hat{T} \to +\infty} \frac{1}{\hat{T}} \int_{-\frac{\tilde{T}}{2}}^{\frac{\tilde{T}}{2}} \int_{-\frac{\tilde{T}}{2}}^{\frac{\tilde{T}}{2}} E\left[\tilde{\xi}_i(t) \tilde{\xi}_j(t')\right] e^{-i\omega(t-t')} dt\, dt' = \delta_{ij} \tag{2.20}$$

From (2.14), we have $\tilde{u}_i(\omega) = \sum_{j=1}^{3} K_{ij}(\omega) \tilde{\xi}_j(\omega); i = 1, 2, 3$ (2.21)

From (2.19) we have $S_{u_i}(\omega) = \sum_{j=1}^{3} \eta_j \left|K_{ij}(\omega)\right|^2 \; ; \; i = 1, 2, 3$ (2.22)

where $K_{ij}(\omega) = \left[M(\omega)\right]^{-1}$

Hence by (2.21) and (2.22), the intensities of fluctuations in the variable u_i ; $i = 1, 2, 3$ are given by

$$\sigma_{u_i}{}^2 = \frac{1}{2\pi} \sum_{j=1}^{3} \int_{-\infty}^{\infty} \eta_j \left|K_{ij}(\omega)\right|^2 d\omega; \; i = 1, 2, 3 \tag{2.23}$$

and from (2.14), (2.15), (2.23) we obtain

$$\sigma_{u_1}{}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\alpha_1\left(X_1{}^2+Y_1^2\right)+\alpha_2\left(X_2{}^2+Y_2{}^2\right)+\alpha_3\left(X_3{}^2+Y_3^2\right)\right]d\omega\right\} \tag{2.24}$$

$$\sigma_{u_2}{}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\alpha_1\left(X_4{}^2+Y_4{}^2\right)+\alpha_2\left(X_5{}^2+Y_5{}^2\right)+\alpha_3\left(X_6{}^2+Y_6^2\right)\right]d\omega\right\} \tag{2.25}$$

$$\sigma_{u_3}{}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\alpha_1\left(X_7{}^2+Y_7{}^2\right)+\alpha_2\left(X_8{}^2+Y_8^2\right)+\alpha_3\left(X_9{}^2+Y_9^2\right)\right]d\omega\right\} \tag{2.26}$$

where $|M(\omega)| = R(\omega)+iI(\omega)$. If we are interested in the dynamics of system (2.1)-(2.3) with either $\alpha_1 = 0$ or $\alpha_2 = 0$ or $\alpha_3 = 0$, then the population variances are as follows.

If $\alpha_1 = 0, \alpha_2 = 0$, then $\sigma_{u_1}{}^2 = \frac{\alpha_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_3{}^2+Y_3^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$; $\sigma_{u_2}{}^2 = \frac{\alpha_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_6{}^2+Y_6{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$;

$$\sigma_{u_3}{}^2 = \frac{\alpha_3}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_9{}^2+Y_9{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega;$$

If $\alpha_2 = 0, \alpha_3 = 0$, then $\sigma_{u_1}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_1{}^2+Y_1^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$; $\sigma_{u_2}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_4{}^2+Y_4{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$;

$$\sigma_{u_3}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_7{}^2+Y_7{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$$

If $\alpha_3 = 0, \alpha_1 = 0$, then $\sigma_{u_1}{}^2 = \frac{\alpha_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_2{}^2+Y_2{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$; $\sigma_{u_2}{}^2 = \frac{\alpha_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_5{}^2+Y_5^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$

$$\sigma_{u_3}{}^2 = \frac{\alpha_2}{2\pi}\int_{-\infty}^{\infty}\frac{\left(X_8{}^2+Y_8{}^2\right)}{R^2(\omega)+I^2(\omega)}d\omega$$

The conditions (2.24)- (2.26) give three varieties of the occupants. The combinations over the genuine line can be evaluated which gives the varieties of the occupants.

III. NUMERICAL SIMULATIONS

Now it is required to approve our investigative discoveries through numerical re-enactments by thinking about the accompanying parameters:

Example 1:

For the parameters $r = 3.5; K = 0.2; a = 0.7; b = 0.9; c = 1.2;\ \alpha = 0.1; \beta = 0.7;\ \eta = 0.2;\ \delta = 0.6;$ $\gamma = 0.36;\ m = 0.2;$ with $\alpha_1 = 1,\ \alpha_2 = 2,\ \alpha_3 = 3$, figure 1(a) represents the variations of populations against time and figure 1(b) represents phase portrait diagram among species.

International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 08, 2020
ISSN: 1475-7192

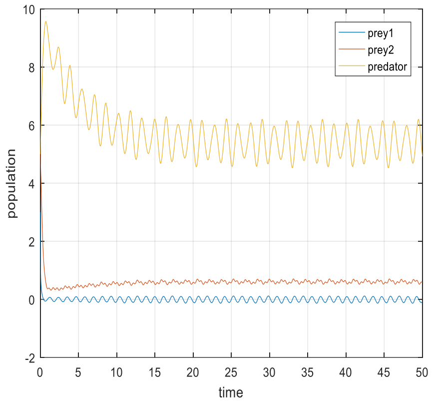

Figure 1(a)

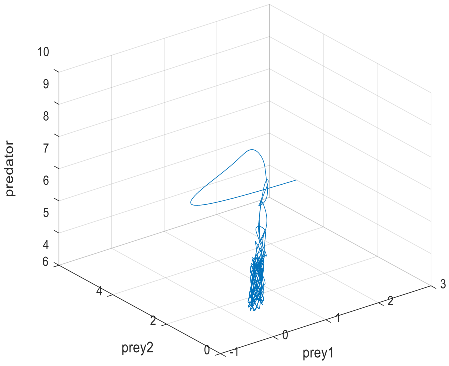

Figure 1(b)

Example 2: For the parameters

$$r = 3.5; K = 0.2; a = 0.7; b = 0.9; c = 1.2; \alpha = 0.1; \beta = 0.7; \eta = 0.2; \ \delta = 0.6; \ \gamma = 0.36; \ m = 0.2;$$

with $\alpha_1 = 10, \ \alpha_2 = 20, \ \alpha_3 = 30$, figure 2(a) represents the variations of populations against time and figure 2(b) represents phase portrait diagram among species.

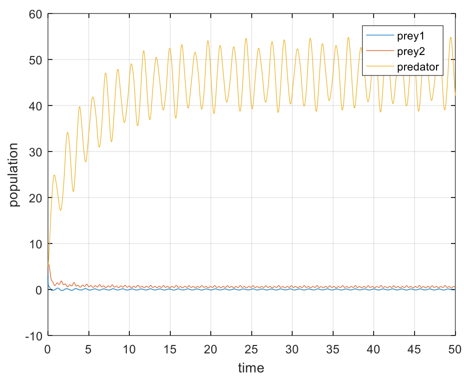

Figure 2(a)

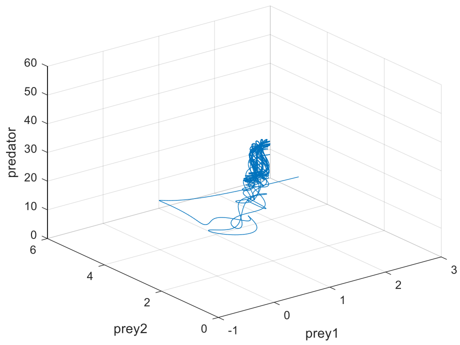

Figure 2(b)

Example 3: For the parameters

$$r = 3.5; K = 0.2; a = 0.7; b = 0.9; c = 1.2; \alpha = 0.1; \beta = 0.7; \eta = 0.2; \ \delta = 0.6; \ \gamma = 0.36; \ m = 0.2;$$

with $\alpha_1 = 30, \ \alpha_2 = 40, \ \alpha_3 = 50$, figure 3(a) represents the variations of populations against time and figure 3(b) represents phase portrait diagram among species.

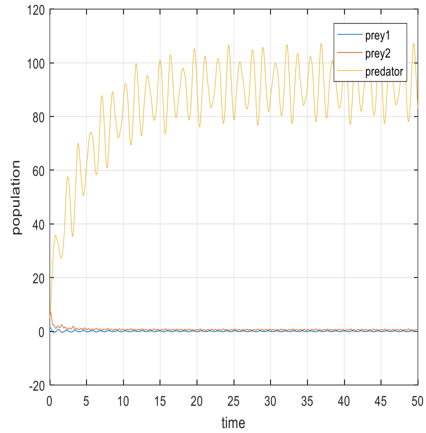

Figure 3(a)

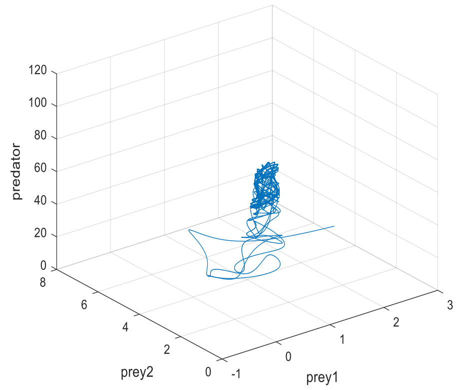

Figure 3(b)

IV. CONCLUDING REMARKS

In this article, we have examined stochastic strength of two prey and one predator around inside consistent state. We additionally infer that the consideration of stochastic irritation makes a critical change in the force of populaces because of progress of responsive parameters causes clamorous elements with low, medium and high differences of motions from figures (1(a), 1(b), 2(a), 2(b), 3(a), 3(b)).

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