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An Imperfect Integrated Production with Advertisement Cost Due to PMC under Fuzzy Systems

[1]V. Kuppulakshmi, [2]C. Sugapriya

Abstract--*The development of imperfect production setup in fuzzy system is proposed in the paper. The demand function is treated as exponential together with inventory. The outcome of the paper mainly concentrates on reducing the total price of the production cycle as the number of cycles increases, this result is compared with the fuzzy environment by using the pentagonal fuzzy number and the graded mean difference method is adopted for defuzzification of the result. Eventually, numerical illustrations are provided for comparing the total cost of the model introduced and the logical analysis are carried out for analyzing the impact of parameter variation on the decision variables.*

Keywords--*Data Mining, Knowledge Discovery, Feature subset, priority instance picking.EPQ, Rework, Inspection cost, Advertising cost, Penalty Maintenance cost,Pentagonal fuzzy number*

I INTRODUCTION

Production system with rework and special sale with price discount has recently become an interesting subject of research. Inspection during the production helps to maintain the quality of items produced. This helps to detect the root of defects instantly and it is useful for any industry that wants to improve production, reduce defect rats, reduce rework and avoid waste.

Central inspection team is responsible for ensuring review of all reports to segregate the damaged goods to the remanufacturing sector. Reviewing the process should be repeated during the production time which results in maintaining the process as well as the quality items. Each and every unit of inventory plays an economic measure in the books of business.

It is important to notice that in real life biodegradable substances that are related agriculture, movie negatives, cosmetic items, and electronic compounds are included. Therefore, they should be sold out as bulk orders with multi shipments with in the pre determined time. If the products remain stagnated, suppliers opt for advertising to promote their products or services. Worldwide, it is estimated that companies spend money in billions for the purpose of advertisements.

The inventory management of the consolidated demand can be a challenge not only from planning horizon, but also from implementation. Physical control of both Stipulations is also tough to handle. Universally e-commerce sales are chosen to be even higher. Profit of the product is directly related to how much or how little

[1]Department of Mathematics, Queen Mary's College, Chennai, India, Email:kuppulakshmi.math2019@gmail.com

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they are able to reduce the holding cost, shipment cost, price discount with low interference to entry and undemanding scalability of an attractive return on investment for many industries.

EPQ model is developed based on imperfect production procedure with the inspection team. This team continuously reviews the damaged products and sends them to a rework. After the production gets over, a multi shipment policy is carried out to supply the demand. Here disjoint delivery is applied to practice.

Even though some items are likely to be unsold, in such a situation suppliers are promoting their products by advertising. Advertisement has a major place for both the producers and the customers. It results in increasing business turn around. Advertisement could be accomplished through different media like TV, newspaper, broadcast, whatsapp, facebook etc. So, it is expected to satisfy a huge number of users. As a result, demand got increased so, backlogging occurs. Due to the imperfect production plan, damaged products are produced and they can be reworked while every set up is completed. Finally, standard items are supplied for the customers. The shortages are fully satisfied by the reworked items. The major areas of this research work are imperfect products, repairing and cost fixing with penalty is discussed. Finally the uncertainty variables like deteriorating rate (□), constant in exponential function of demand (α), and percentage of reworkable products (ϕ) are treated as pentagonal fuzzy numbers. The graded mean representation method is adopted for defuzzification of the result.

II RELATED WORK

C. Sugapriya (2008) considered the EPQ model for non-instantly degrading goods requiring price discount and the principle of allowable delay in payments was addressed. Shortages during development process are not allowed. For the proposed back orders for a single product, Leopoldo Eduardo Cárdenas –Barrón et al. (2009) enhanced the quantity of economic production in a single manufacturing system generating and reworked in the same process along with the faulty reliability modules. Takeshikoide et al. (2009) discussed the strategies of increasing the day's revenue in the monopolist firm. This paper also deals with the discount pricing for unsold products using reference effects and inventory constraint. Ata Allah the EPQ model developed by Taleizadeh and et al (2010) is designed to calculate the overall anticipated annual cost by optimizing the duration of the period, back-order and reworked items. He introduced the concept of service level constraint and production capacity to prove the model to be convex.

A.A. Taleizadeh et al (2010) investigated a multiproduct production quantity using one machine with the limited production capacity. This paper started with single product case and extended the concept to multiproduct case with the reworked and scrapped items. In the same year A. A.Taleizadeh et al (2010) derived The EPQ model has a single machine with limited capacity, but there is no rework, but there is a shortage. It is assumed that all defective items have been scrapped.

C. Sugapriya et al (2011) followed a model to minimize the cost using production rate and demand rate which remains constant. And also two levels of trade credit are discussed and allowed the price discount for deteriorating items. A.A. Taleizadeh and et al (2011) developed Multiproduct EPQ model using a single computer and backorders but the idea of immediate rework without scrapped products left was implemented in this article. Also he compared the result with rework and the result without rework. A.A. Taleizadeh and et al (2012) took a single product EPQ model which is later modified as a multiproduct case. During the rework some scrap items are also produced and then it is disposed in the last part of the cycle and it includes disposal cost included. A. Nagoor Gani et al (2013) developed the fuzzy optimal ordering strategy in inventory model. Maximized cost function is defined in this paper retailer's gross profit per unit time as fuzzy numbers.

Vinodkumar Mishra et al (2013) gave an EOQ inventory model for time varying holding cost and time dependent demand. Here deterioration is time proportional. To minimize the total inventory cost is the main aim of this paper. A. A. Taleizadeh et al. (2013) developed reworked, multi-product ion EPQ platform and lost sales. Within this paper a new idea is explored to reduce the overall annual expense according to the level of operation and budget constraints.

A.A. Taleizadeh et al (2013) extended the classical EOQ model for perishable items with temporary discount in unit purchasing cost and shortage quantities are determined using closed – form equations. S. Priyan et al (2014) considered EOQ inventory model with two types of fuzzy numbers, triangular and trapezoidal. Classical technique of differential optimisation is used to find the model's optimum solution.

Leopoldo eduardo cardenas et al. (2015) developed the previous work with the same shipment for inventory but presented the optimizing procedure of the replenishment of a huge size which includs delivery cost. S. Priyan et al. (2015) designed the deterioration rate as the triangular fuzzy number and the setup cost is reduced through extra investment. Finally, proved that the optimal total cost is minimized. K.V. Geetha et al. (2016) discussed with EOQ model for non – instantaneous deteriorating items with salvage value parameter allowing shortage which leads to lost sale. A.A. Taleizadeh et al. (2016) explained the EPQ model with three level supply chain for multiproduct and also the rework. In this paper multiple raw materials and multiple products are assumed. Michael et al. (2015) classified the fuzzy logic due to its nature and derived many definitions and defuzzification methods for research purpose. S. K. Indrajit Singh et al. (2016) discussed the fuzzy inventory model for both crisp case and fuzzy environment. In this model all parameters are assumed to be a hexagonal fuzzy numbers.

C. Sugapriya (2017) deals with EPQ model for a single product subjected to exponential distribution using permissible delay in payments is discussed. In addition, price discount is allotted for deteriorating items. C. Sugapriya (2017) formulated an EPQ model for deteriorating items with exponential distribution in an infinite planning horizon which provides also price discount for the deteriorating items. R. Uthayakumar et al. (2017) considered productivity and rework with deterioration as the major concern. In this paper one cycle has n production setups and only one rework setup is considered. Two cases are discussed (i) stack dependent demand and (ii) exponential demand. W. Q. Zhou et al (2017) dealt with inventory control model of the remanufacturing reverse logistics. In this paper he established the multi - echelon inventory relationship between the supplier and the customer through the manufacturer.

T. Sekar et al (2018) focused a production inventory model for single vendor single buyer with no breakdown during the production. The total relevant cost for vendor and buyer is demonstrated in this model. G. S. Mahapatra et al. (2019) develops a fuzzy inventory model allowing trade credit and permissible delay in payments. Symmetric triangular fuzzy number is - integral value is used. Rituparna Pakhira et al (2019) discussed the EOQ model with memory effect property under fuzzy environment. The result is more feasible by using the graded integration method for defuzzification. In 1965 Zadeh launched the notations of a fuzzy set. Definitions are given below:

Definition 1.1

A fuzzy set $\tilde{\Delta}$ in X is represented as a set of ordered pairs as given below

$$\tilde{\Delta}=\{(N,\mu_{\tilde{A}}(N)) : N \in X\} \quad \text{where}$$

$$\mu_{\tilde{\Delta}}: X \to [0,1]$$

If X is a finite set then $\tilde{\Delta}$ is presented as $\sum_{N\in X} \mu_{\tilde{\Delta}}(N)/N$

If X is a countable set then $\tilde{\Delta}$ is presented as $\sum_{i=0}^{\infty} \mu_{\tilde{\Delta}}(N_i)/N_i$

If X is a continuous case then $\tilde{\Delta}$ is presented as integral $\int_X \mu_{\tilde{\Delta}}(N)\, dn$

Definition 1.2

A fuzzy set $\tilde{\Delta} = \{(X, \mu_{\tilde{\Delta}}(X))\} \subseteq$ is convex fuzzy set if all $\tilde{\Delta_x}$ are convex set that is for each element $u\in\Delta_\propto$ and $v\in\Delta_\propto$ for every $\propto\in[0,1]$. $u\lambda + (1-\lambda) v, \forall\ \lambda\in[0,1]$. On the otherwise fuzzy set is called non – convex fuzzy set [11].

Definition 1.3

A fuzzy set $[u_\propto, v_\propto]$ where $0\le \propto \le 1$ and $a< b$ determined on R is termed as fuzzy interval when its membership function is given by

$$\mu_{([a_\propto, b_\propto])}=\begin{cases} \propto, & u \le x \le v \\ 0, & \text{or else} \end{cases}$$

Definition 1.4

A fuzzy number (FN) $\tilde{\Delta}$ = (u, v, w) where u<v<w and determined on R is termed as triangular fuzzy number when function of membership is

$$\mu_{\tilde{\Delta}}(X)=\begin{cases} \frac{x-u}{v-u}, & u\le x\le v \\ \frac{w-x}{w-v}, & v\le x\le w \\ 0, & \text{otherwise} \end{cases}$$

When u = v = w, we have fuzzy points (w, w, w) = w the group of all triangular fuzzy numbers on R is determined as,

$$F_N\ \{(u,v,w): u<v<w\ \forall u,v,w \in R\}$$

The $\propto$ - cut of $\tilde{\Delta}=(u,v,w)\in F_N$, $0\le \propto \le 1$, is $\Delta(\propto)=[\Delta_L(\propto),\Delta_R(\propto)]$.

Where $\Delta_L(\propto)=u+(v-u)\propto$ and $\Delta_R(\propto)=w-(w-v)\propto$ are the left and the right end points of $\Delta(\propto)$.

Definition 1.5

A pentagonal fuzzy number $\tilde{\Delta}$ = (a, b, c, d, e,) is indicated with performance of membership $\mu_{(\check{A})}$.

$$\mu_{(\check{\Delta})}=\begin{cases} L_1(x)=\frac{1}{2}\left(\frac{x-a}{b-a}\right), & a\le x \le b \\ L_2(x)=\left(\frac{x-b}{c-d}\right), & b\le x \le c \\ 1, & x=c \\ R_1(x)=\left(\frac{d-x}{d-c}\right), & c\le x \le d \\ R_2(x)=\left(\frac{e-x}{e-d}\right), & d\le x \le e \\ 0, & \text{otherwise} \end{cases}$$

The $\propto$ - cut on $\tilde{\Delta}$ = (a, b, c, d, e,) , $0\le \propto \le 1$ is $\Delta(\propto) = [\Delta_L(\propto),\Delta_R(\propto)]$.

where

$$\Delta_{(L_1)}(\propto)=a+(b-a)\propto= L_1^{-1}(\propto)$$

$$\Delta_{(L_2)}(\propto)=b+(c-b)\propto= L_2^{-1}(\propto)$$

$$\Delta_{(R_1)}(\propto)=d-(d-c)\propto= R_1^{-1}(\propto)$$

$\Delta_{R_2}(\propto)=e-(e-d)\propto= R_2^{-1}(\propto)$

Hence,

$L^{-1}(\propto)=(L_1^{-1}(\propto)+L_2^{-1}(\propto))/2=(a+b+(c-a)\propto)/2$

$R^{-1}(\propto)=(R_1^{-1}(\propto)+R_2^{-1}(\propto))/2=(d+e-(e-c)\propto)/2$

Definition 1.6

If $\tilde{\Delta} = (u, v, w, x, y)$ is a pentagonal fuzzy number and GMRM of $\tilde{\Delta}$ is derived as

$P(\tilde{\Delta})=1/12[u+3v+4w+3x+y]$

Fig.1 Inventory process in two production setup and one rework

Assumptions:

Demand is exponential and stock dependent.

Shortages are allowed but they are fully satisfied by the reworked products.

Production of standard products must be more than the demand.

Damaged goods are continuously collected by the inspection team during the production period.

No breakdown occurs during the process.

During the shipment period $[0,T_2]$ products are sent to the customers at regular interval of period.

Advertisement cost is linearly depending on time $(a+b(\tau))$ during the period $[\mu,T_2]$. where a, b are constant.

During the shipment period inventory level remains stable so, penalty maintenance cost is allowed in this period.

Rework rate is constant.

All imperfect products are repairable at completion of each production.

Finally, deteriorating rate (ω), constant in exponential function of demand (α), and percentage of reworkable products (ϕ) are treated as pentagonal fuzzy numbers.

Notations

- Operational inventory level in manufacturing period
- Operational inventory level in non production period
- Operational inventory level in repairing time

I_{r1} - Retrievable inventory in production time

I_{r2} - Retrievable inventory in non manufacturing duration

I_{r3} - Retrievable inventory in rework time

- Total operational inventory in manufacturing time
- Net operational inventory in non manufacturing time
- Net operational inventory in repairing time

$I_{\tau r1}$ - Net retrievable inventory in manufacturing time

〖I_τr〗_2 - Net retrievable inventory in non manufacturing time

〖I_τr〗_3 - Net retrievable inventory in repairing time

- Net retrievable inventory in non manufacturing time

I_max - Maximum inventory level of retrievable products in manufacturing setup

- Maximum inventory level of retrievable products in repairing procedure

- Rework process rate (per unit)

- Manufacturing setup (cost/setup)

- Repairing production setup (cost/setup)

- Operational products holding price (per unit)

- Retrievable item holding price (per unit)

- Deteriorating rate (per unit)

P - Production rate

K - Manufacturing price per unit item

c - Shortage price per unit time

- Time period at which the deterioration of the product starts

ϕ - Percentage of reworkable products

r - Price discount per unit time

SHC - Shortage cost in rework period

PD - Price discount in rework period

PMC - Penalty maintenance cost in shipment period

IPC - Inspection cost during production period

〖Ad〗_c - Advertisement cost during shipment period

〖Ad〗_r - Advertisement rate during shipment period

- Total deteriorating unit

- Total time in production period

- Total time in shipment period

- Total time in rework period

T - Total cycle time

n - Number of production setup in one cycle

SPC - Shipment cost

m - Number of shipments

Problem Formulation

At the time τ=0 the production starts and the inventory is zero at this time. After some time damaged products are produced, and these damaged products are continuously reviewed by inspection team and the damaged products are collected separately during the production time for rework. During the period [0,T_1] production and supply start simultaneously and the inventory level reaches maximum I_{max}at the time $\tau=T_1$. The production ends at this time and on hand inventory reduces by deterioration as well as by supply to customers. The supply of products takes place during the period [$0,T_2$], and all good quality items are delivered (time units) at constant interval of period between each shipment. $(T_2=\bar{\tau_1}+\bar{\tau_2}+\cdots+\bar{\tau_n})$

During the period [μ,T_2] there is no supply, inventory level remains same. So, the manufacture giving advertisement for the product.

After advertisement, the demand of product increases and the back logging occurs. During this [$0,T_3$] period the rework production starts, and it satisfies the demand as well as shortages. During this rework period inventory level reachesI_{maxRR}.

Manufacturing Inventory level

Exponential and stock depended demand $D=\propto e^{\beta\tau}+\beta I(\tau)$ is assumed for this manufacturing model. Were $\propto>0$ and $0<\beta<1$.

The stock level of producing products grows gradually from the period

$[0,T_1]$, and the damaged products are collected separately. And so, the stock level in manufacturing time is given as

$$\frac{dI_{p_1}(\tau_1)}{d\tau_1}+\omega I_{p_1}(\tau_1)=(1-\phi)P-\propto e^{\beta\tau_1}+\beta I_{p_1}(\tau_1), \quad 0\le\tau_1\le T_1 \tag{1}$$

with the condition that $I_{p_1}(0)=0$

The stock level is reduced in the non production together with shipment in $[0,T_2]$ is given as,

$$\frac{dI_{p_2}(\tau_2)}{d\tau_2}+\omega I_{p_2}(\tau_2)=-\propto e^{\beta\tau_2}+\beta I_{p_2}(\tau_2) \quad 0\le\tau_2\le T_2 \tag{2}$$

with the condition that $I_{p_2}(0)=0$

The stock level at the time of rework $[0,T_3]$ is given as

$$\frac{dI_{p_3}(\tau_3)}{d\tau_3}+\omega I_{p_3}(\tau_3)=p_r-\propto e^{\beta\tau_3}+\beta I_{p_3}(\tau_3) \quad 0\le\tau_3\le T_3 \tag{3}$$

with the condition that $I_{p_3}(0)=0$

The Manufacturing model with the reworkable items is derived by solving equation (1) using initial condition and the operational inventory level is obtained in the production period $\tau_1$during the interval $[0,T_1]$ as

$$\frac{dI_{p_1}(\tau_1)}{d\tau_1}+\omega I_{p_1}(\tau_1)=(1-\phi)P-(\propto e^{\beta\tau_1}+\beta I_{p_1}(\tau_1)) \tag{4}$$

$$I_{p_1}(\tau_1)=\frac{(1-\phi)p}{\omega+\beta}\left(1-e^{-(\omega+\beta)\tau_1}\right)-\frac{\alpha}{\omega+2\beta}\left(e^{\beta\tau_1}-e^{-(\omega+\beta)\tau_1}\right) \tag{5}$$

By integrating the manufacturing stock level in the manufacturing time with time τ_1 gives the total inventory in a production period.

$$I_{\tau p_1}(\tau_1)=\int_0^{T_1}\left[\left(\frac{(1-\phi)p}{\omega+\beta}\right)\left(1-e^{-(\omega+\beta)\tau_1}\right)d\tau_1-\int_0^{T_1}\left(\frac{\alpha}{\omega+2\beta}\right)\left(e^{\beta\tau_1}-e^{-(\omega+\beta)\tau_1}\right)d\tau_1\right] \quad (6)$$

Considering the equation (IA.1), the third and the higher powers are neglected for small values of $\frac{(\omega)^3 T_1^3}{3!}\ll 1$

$$I_{\tau p_1}(\tau_1)=\frac{(1-\phi)p-\propto}{2}T_1^2 \quad (7)$$

The operational inventory level is obtained in non production period τ_2 during the interval $[0,T_2]$ as

$$\frac{d I_{p_2}(\tau_2)}{d\tau_2}+\omega I_{p_2}(\tau_2)=-\left(\alpha e^{\beta\tau_2}+\beta I_{p_2}(\tau_2)\right) \quad (8)$$

By integrating non production inventory level in the shipment period with time gives the net stock level in a non manufacturing time

$$I_{p_2}(\tau_2)=\int_0^{T_2}\left(\frac{\alpha}{\omega+\beta}\right)\left(e^{(2\beta+\omega)(T_2-\tau_2)}-1\right)d\tau_2 \quad (9)$$

$$I_{\tau p_2}(\tau_2)=\frac{\propto}{2}T_2^2 \quad (10)$$

when $I_{p_1}=I_{p_2}$, $\tau_1=T_1$ an $\tau_2=0$ then the following is obtained

$$\left(\frac{(1-\phi)p}{\omega+\beta}\right)\left(1-e^{-(\omega+\beta)\tau_1}\right)-\left(\frac{\alpha}{\omega+2\beta}\right)\left(e^{\beta\tau_1}-e^{-(\omega+\beta)\tau_1}\right)=\left(\frac{\alpha}{\omega+2\beta}\right)\left(e^{(2\beta+\omega)T_2}-1\right) \quad (11)$$

then

$$T_2=\left(\frac{(1-\phi)p}{\alpha}\right)\left[T_1-\frac{(\omega+\beta)T_1^2}{2}\right]-\left[T_1-\frac{\omega T_1^2}{2}\right] \quad (12)$$

The stock level in repairing time is given below

$$\frac{d I_{p_3}(\tau_3)}{d\tau_3}+\omega I_{p_3}(\tau_3)=p_r-\left(\alpha e^{\beta\tau_3}+\beta I_{p_3}(\tau_3)\right) \quad (13)$$

$$I_{\tau p_3}(\tau_3)=\frac{p_r-\alpha}{2}T_3^2 \quad (14)$$

At the time of production, some items are produced with defects and they are stored. Defective items are inspected by the team and those items are collected for rework process; these items can be calculated as

$$\frac{d I_{r_1}(\tau_{r_1})}{d\tau_{r_1}}+\omega I_{r_1}(\tau_{r_1})=\phi P,\quad 0\le \tau_{r_1}\le T_1 \text{ with } I_{r_1}(0)=0 \quad (15)$$

Solving the above it is derived as

$$I_{\tau r_1}(\tau_{r_1})=\frac{\phi P}{2}T_1^2 \quad (16)$$

At the Initial stage, the recoverable items are denoted by I_{maxR}

$$I_{max}=\frac{(\phi P)}{\omega}\left(1-e^{-\omega T_1}\right) \quad (17)$$

Considering the equation (IA.2), the third and the higher powers are neglected for small values of $\frac{\omega^3 T_1^3}{2}\ll 1$

$$I_{max}=\phi P\left(T_1-\frac{\omega T_1^2}{2}\right) \quad (18)$$

During the shipment period the recoverable items are

$$\frac{dI_{r_2}(\tau_{r_2})}{d\tau_{r_2}}+\omega I_{r_2}(\tau_{r_2})=0, \text{ with } \tau_{r_2}\le T_2, \text{ with } I_{r_2}(0)=0 \quad (19)$$

initially,

$$I_{\tau r_2}(0)=I_{max} \quad (20)$$

The total stock level of retrievable products in the non manufacturing time is

$$I_{\tau r_2}(\tau_{r_2})=\int_{\tau_{r_2}=0}^{(n-1)T_1+nT_2} I_{max}\, e^{-\omega\tau_{r_2}}\, d\tau_{r_2} \quad (21)$$

$$I_{V1}=\sum_{s=1}^{n}\int_{\tau_{r_2}=0}^{(s-1)T_1+sT_2} I_{max}\, e^{-\omega\tau_{r_2}}\, d\tau_{r_2} \quad (22)$$

$$I_{V1}=\sum_{s=1}^{n} I_{max}\left[\left((s-1)T_1+sT_2\right)-\frac{\omega\left((s-1)T_1+sT_2\right)^2}{2}\right] \quad (23)$$

At the completion of one manufacturing cycle, the stock level attains maximum

$$I_{maxRR}=\sum_{s=1}^{n} I_{max}\left[\left(1-\omega\left((s-1)T_1+sT_2\right)+\frac{\omega^2\left((s-1)T_1+sT_2\right)^2}{2}\right)\right] \quad (24)$$

The stock level of retrievable products in rework

$$\frac{dI_{r_3}(\tau_{r_3})}{d\tau_{r_3}}+\omega I_{r_3}(\tau_{r_3})=-P_r, \text{ with } \tau_{r_3}\le T_3, \text{ with } I_{r_3}(0)=0 \quad (25)$$

$$I_{\tau r_3}(\tau_{r_3})=\frac{P_r}{\omega}\left(e^{\omega(T_3-\tau_{r_3})}-1\right) \quad (26)$$

The total recoverable items are

$$I_{\tau r_3}=\frac{P_r T_3^2}{2} \quad (27)$$

Equation (27) can be written as

$$I_{maxRR}=\frac{P_r}{\omega}\left(e^{\omega T_3}-1\right) \quad (28)$$

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T_3=I_maxRR/P_r

(29)

The total operational and the recoverable inventory are given as

ToTI=nI_(〖τp〗_1)+ 〖nI〗_(〖τp〗_2)+I_(〖τp〗_3)

(30)

TRI=nI_(〖τr〗_1)+I_V1+I_(〖τr〗_3)

(31)

θ_T=[nϕPT_1+P_r T_3]-[n(α+βI_(〖τp〗_1))T_1+n(α+βI_(〖τp〗_2))T_2+(α+βI_(〖τp〗_3))T_3] (32)

The Total cost of the manufacturing cycle

The inspection amount during the production period is given as

INC=(I_c I_r)/T_1 ∫_0^(T_1)▒〖Pdτ_1 〗

(33)

INC=I_c I_r P (34)

During the non production time [μ,T_2]the demand remains stagnated so the maintenance cost turns into penalty maintenance cost which is given as

PMC=π(T_2-μ)αe^(β〖(T〗_2-μ))+βI_(〖τp〗_2) (T_2-μ) where π=3.141

(35)

During the shipment period, as supply remains stagnated, the supplier wants to promote the product by advertisement and the advertisement cost is given as

ADC=(Ad_c Ad_r)/T_2 ∫_μ^(T_2)▒〖(a+bτ_2) dτ_2 〗 where a and b are constants

(36)

ADC =〖Ad〗_c 〖Ad〗_r [a+(bT_2)/2-aμ/T_2 -(bμ^2)/〖2T〗_2]

(37)

Price discount is allowed for bulk orders during the time[μ,T_2] is given as

PD=kr/T_2 ∫_μ^(T_2)▒〖αe^(βτ_2)+βI_(〖τp〗_2) (τ_2) dτ_2 〗

(38)

PD=kr/T_2 [(αe^(βT_2))/β+βI_(〖τp〗_2) (τ_2) T_2-(αe^βμ)/β-βI_(〖τp〗_2) (τ_2)μ]

(39)

The shortage cost is calculated during the rework period is given as

SHC=c∫_0^(T_3)▒〖-αe^(βτ_3)+βI_(〖τp〗_3) (τ_3)dτ_3 〗

(40)

SHC=-c[(αe^(βT_3))/β+βI_(〖τp〗_3) (τ_3) T_3]

(41)

The total cost is given as

$$TC=(nA_P+A_r+H_p(TOTI)+H_r(TRI)+\omega\theta_T+INC+PMC+ADC+PD+SHC+m(SPC))/(n(T_1+T_2)+T_3) \quad (42)$$

Algorithm

Considering the production time alone to get T_1.so, in equation (42) we are considering the cost functions only involving the time period T_1.

Step 1: Substitute n = 1 in the equation (42) differentiate with the condition $\partial TC/(\partial T_1)=0$

Step 2: Find the value for equation (42)

Step 3: Substitute in equation (42) to get TC

Numerical illustration

The parameters for this inventory models are P = \$1000/unit, P_r=\$300, ϕ= 0.06, A_P=\$6/unit, A_r=\$1.5/unit, H_P=\$3/unit, k = 5, H_r=\$0.6/unit,α = 102, π = 3.414, r = 0.1, μ = 0.01, β=0.05, a=0.1, b=5, c = 0.15, Ad_c=\$0.03/unit, Ad_r=0.15, I_c=\$1/unit, I_r=3, m = 5, SPC = 100, and m = 5. Iterating the values of n, the optimal solution is found as a nearby value of n

Finally, the value for TC = 12138 for n = 1 and T_1=0.03 is obtained.

Table 1: Calculation of optimal total cost in exponential and the inventory dependent demand by increasing the number of cycles, the total cost is decreasing and it is given as

N	1	2	3	4	5	6	7	8	9	10
Total cost	12138	6099	4087	3082	2478	2076	1789	1574	1406	1273

Table 2: Calculation of optimal total cost in exponential and the inventory dependent demand by increasing the number of cycles and the production time T_1 ,the total cost is decreasing and it is given as

N	1	2	3	4	5	6	7	8	9	10
T_1	0.0300	0.0378	0.0456	0.0533	0.0611	0.0689	0.0767	0.0844	0.0922	0.1000
Total cost	12138	4895	2763	18211	1315	1013	819	690	599	534

In case of uncertainty, it is not possible to change all the parameters as fuzzy numbers. Here, the deteriorating rate (ω), constant in exponential function of demand (α) and the percentage of reworkable products (ϕ) are treated as pentagonal fuzzy numbers and they are given as,

Let $\breve{\omega}=(0.04,0.06,0.08,0.1,0.12)$, $\breve{\phi}=(0.054,0.057,0.06,0.063,0.066)$ and

$\breve{\alpha} = (92,97,102,107,112)$. The total cost of the production cycle under fuzzy systems is given as

$$\breve{TC} = \frac{nA_P + A_r + H_p(\breve{TOTI}) + H_r(\breve{TRI}) + \breve{\omega}\breve{\theta}_T + INC + \breve{PMC} + ADC + \breve{PD} + \breve{SHC} + m(SPC)}{n(T_1 + \breve{T}_2) + \breve{T}_3}$$

This can be defizzified by GMRM which is given,

$$\breve{TC} = \frac{1}{12}\left[\breve{TC}_1 + 3\breve{TC}_2 + 4\breve{TC}_3 + 3\breve{TC}_4 + \breve{TC}_5\right]$$

$$\breve{TC}_1 = \frac{nA_P + A_r + H_p(\breve{TOTI})_1 + H_r(\breve{TRI})_1 + \breve{\omega}_1\breve{\theta}_{T1} + INC + \breve{PMC}_1 + ADC + \breve{PD}_1 + \breve{SHC}_1 + m(SPC)}{n(T_1 + (\breve{T}_2)_1) + (\breve{T}_3)_1}$$

$$\breve{TC}_2 = \frac{nA_P + A_r + H_p(\breve{TOTI})_2 + H_r(\breve{TRI})_2 + \breve{\omega}_2\breve{\theta}_{T2} + INC + \breve{PMC}_2 + ADC + \breve{PD}_2 + \breve{SHC}_2 + m(SPC)}{n(T_1 + (\breve{T}_2)_2) + (\breve{T}_3)_2}$$

$$\breve{TC}_3 = \frac{nA_P + A_r + H_p(\breve{TOTI})_3 + H_r(\breve{TRI})_3 + \breve{\omega}_3\breve{\theta}_{T3} + INC + \breve{PMC}_3 + ADC + \breve{PD}_3 + \breve{SHC}_3 + m(SPC)}{n(T_1 + (\breve{T}_2)_3) + (\breve{T}_3)_3}$$

$$\breve{TC}_4 = \frac{nA_P + A_r + H_p(\breve{TOTI})_4 + H_r(\breve{TRI})_4 + \breve{\omega}_4\breve{\theta}_{T4} + INC + \breve{PMC}_4 + ADC + \breve{PD}_4 + \breve{SHC}_4 + m(SPC)}{n(T_1 + (\breve{T}_2)_4) + (\breve{T}_3)_4}$$

$$\breve{TC}_5 = \frac{nA_P + A_r + H_p(\breve{TOTI})_5 + H_r(\breve{TRI})_5 + \breve{\omega}_5\breve{\theta}_{T5} + INC + \breve{PMC}_5 + ADC + \breve{PD}_5 + \breve{SHC}_5 + m(SPC)}{n(T_1 + (\breve{T}_2)_5) + (\breve{T}_3)_5}$$

The result of fuzzy model is derived by GMRM method which is given as,

$\breve{TC}^* = \$12137$ for n = 1 and T_1 = 0.03

III CONCLUSION

This research shows that, the fuzzy EPQ model with exponential and time dependent demand gives more feasible result while comparing the result with the crisp case. Here, the deteriorating rate (ω), constant in exponential function of demand (α), and percentage of rework able products (ϕ) are treated as pentagonal fuzzy numbers. The graded mean representation method is applied for defuzzification of the result. Sensitive analysis is carried out through which the proportion of variation in the total cost is determined. The research can be developed further by assuming the cost parameters as pentagonal fuzzy numbers

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