Use of Lévy's Model to Simulate Stock Dividends Actual Estimations of some Iraqi Banks

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Abstract: *In this study, one of the models of random contingent processes was studied, which is one of the Lévy models, based on the Brownian subordinate. It was relied on the so-called Normal Inverse Gaussian (NIG). This study aims at estimating the parameters of this model using the moments and maximum likelihood methods and then employing these estimations for the parameters in studying stock dividends and knowing the market efficiency of both the United Bank and the North Bank whose data were collected from Iraq Stock Exchange. As well as the use of simulation method to simulate the applied side with the different assumed cases. The simulation results also indicated that the increasing value of kurtosis parameter and the decreasing skewness parameter in the NIG model resulted in a decrease in the large fluctuations, especially when increasing the sizes of the samples. The decrease in volatility parameter was better than its increase for the North Bank, unlike the United Bank, where the increase of volatility had an impact on the reduction of the MASE.*

Keywords: *Lévy's Model, Stock Dividends, Iraqi Banks*

I. INTRODUCTION

With the expansion of economic activity, it has become necessary to establish large financial markets. As the stock market is one of the most important fields of investment that allows large and small investors to make profits, this requires sufficient information about securities traded. Which have a role in reducing risks and lowering them to the lowest level.

The analysis of stock markets, securities and all investment alternatives in terms of the expected dividend of these investments and the potential risks that investors may face the basis of the investment process.

The dividend is one of the key indicators for the purpose of investment in the shares of companies and it refers to an appropriate measure of the trade-off between stocks or reward obtained by the investor in compensation for the waiting period and the potential risk of operating funds in the stocks.

It is worth mentioning that there is a difference in prices or in the stock markets. Therefore, it is necessary to know the statistical characteristics of price variable for different types of securities or dividend assets. These characteristics can be described in terms of the fact that they are not linked to the dividend assets, as the link is often weak but for the very small units (Moments), link may be taken.

The statistical characteristics of the dividends are also heavy ruminants due to the increase in daily repetitions of dividends as well as the presence or absence of symmetry. The properties of the statistical estimation are stable and periodic. Therefore, because of the statistical properties in the dividend assets, it is necessary to resort to other methods or models that possess this type of properties, and one of these models called Lévy process, which is one of the

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contingent process types, which is characterized by its ability to model the behavior of the variable imposed in the dividends.

It should be noted that the process or model Lévy refers to being a measure of the economic effectiveness, being able to describe the actual reality of the stock markets movement studied when using models refer to the Brownian movement (a special case of the Lévy process by assuming that the random variable is distributed naturally).

As well as the possibility of this type of models or processes to study fluctuations in the markets or stocks when there are sudden jumps in those markets, making them more effective for the dynamics of securities represented in the stock markets and pricing options. With its suitability of these types of models to distribute the assets of the dividends.

II. BUILDING NIG FOR THE LÉVY PROCESS BASED ON THE BROWNIAN SUBORDINATE:

The basis for the construction of Lévy model based on the partial coordinate of the Brownian motion (which is similar to Lévy processes with a more flexible approach compared to the similar Lévy processes being composed of partial coordinate with the Brownian motion).

The existence of both so called partial coordinate S_t, (which represents Lévy processes when $\leq X_s\ X_t$ is realized for each $t \geq s$ being $t \geq 0$), as well as the Brownian motion W_t, which is dependent from the partial coordinate S_t, as Lévy's model can be summarized based on the Brownian subordinate according to the following formula:

1) … $X_t = \sigma W_{S_t} + M\, S_t$

Represents diffusion σ

M represents drift

$W_{(st)}$ represents the Brownian Motion of the partial coordinates, and when S_t in the equation 1 refers to the inverse Gaussian (IG), X_t represents the process or model of NIG. Therefore, by rewriting equation 1 it becomes:

2)… $X_t^{NIG} = M X_t^{IG} + \sigma\, W_{X_t^{IG}}$

X_t^{NIG} represents NIG process or model, and the distribution of the model is

NIG ($\alpha, \beta, \delta_t, M_t$)

And X_t^{IG} represents the Inverse Gaussian with the assumption that:

$= \delta\sqrt{\alpha^2 - \beta^2}$) $\eta = 1$, γ

As for W, it refers to Wiener process or standard Brownian motion.

By returning to equation 2, X_t^{NIG} can be written as follows:

3)… $X_t^{NIG} = \beta\delta^2 X_t^{IG} + \delta W_{X_t^{IG}}$

Supposing that:

$$\beta = \frac{M}{\sigma^2} \quad , \quad \alpha^2 = \frac{\gamma^2}{\sigma^2} + \frac{M^2}{\sigma^4} \quad , \quad \gamma = \delta\sqrt{\alpha^2 - \beta^2} \quad , \quad \delta = \sigma$$

And the potential distribution of the model is NIG (α , , δ_t)

Therefore, by depending on the previous assumptions we can write the equation 3 X_t^{NIG} as follows:

4)… $X_t^{NIG} = \beta\delta^2 X_t^{IG} + \delta W_{X_t^{IG}} + M_t$

And the potential distribution of the model in the equation above is NIG (α , β , δt ,Mt)

And the density function X for NIG is:

5)… $$f\left(X_t^{NIG}(x)\right)=\frac{\alpha\delta}{\pi}\exp\left[\delta t\sqrt{\alpha^2-\beta^2}+\beta(x-Mt)\right]\times\frac{k_1\left(\alpha\sqrt{\delta^2t^2+(x-Mt)^2}\right)}{\sqrt{\delta^2t^2+(x-Mt)^2}}$$

Provided that

$\alpha > 0$, $-\alpha < \beta < \alpha - 1$, $\delta > 0$

(.) K_1 represents the modified Bessel function of the second type

$$K_\lambda(u)=\frac{1}{2}\int_0^1 u^{\lambda-1}e^{\frac{-1}{2}\left(z(u+u^{-1})\right)}du$$

λ represents Bessel function of the first or the second type

α refers to kurtosis parameter

β refers to skewness parameter

δ refers to Scale parameter which is similar to Standard deviation parameter (σ) in the normal distribution and it represents dividends spread scale.

M refers to location parameter which is often imposed as being zero (M= 0) because it does not affect the pricing.

1- Properties of Normal Inverse Gaussian for Lévy process (NIG- Lévy)

NIG- Lévy model is characterized by some properties such as:

1- This model is characterized by being infinitely divisible, as NIG distribution considered Lévy process or Lévy model and vice versa.

2- By relying on Lévy khintchine formula for NIG- Lévy model which is represented by:

6)… $$\Psi(Z)=\left(-\delta t\left(\sqrt{\alpha^2-(\beta+Z)^2}-\sqrt{\alpha^2-\beta^2}\right)+MZ\right)$$

Then Moments of NIG- Lévy model was found, that is:

7)… $$EX_t^{NIG}=Mt+\frac{\delta t\beta}{\sqrt{\alpha^2-\beta^2}}$$

8)… $$Var(X_t^{NIG})=\frac{\delta t\,\alpha^2}{\left(\alpha^2-\beta^2\right)^{3/2}}$$

9)… $$skew(X_t^{NIG})=\frac{3\beta}{\alpha\sqrt{\delta t}\left(\alpha^2-\beta^2\right)^{1/4}}$$

10)… $$kurt=3\left(1+\frac{\alpha^2+4\beta^2}{\delta t\,\alpha^2\sqrt{\alpha^2-\beta^2}}\right)$$

3- The triple of the NIG- Lévy model mentioned in equation 3 can be written as follows:

A- Drift for NIG- Lévy model

11)… $$\gamma^{NIG}=\frac{2\delta\alpha}{\pi}\int_0^1 Sinh\ (\beta x)\,k_1(\alpha x)\,dx$$

And that K1 represents Bessel function of the second type

$$Sinh\ (\beta x) = \frac{1}{2}\left(e^{\beta x} - e^{-\beta x}\right)$$

B- Lévy scale

12)… $$V^{NIG}(x) = \frac{\delta\alpha e^{\beta x}\left(\alpha | x |\right)}{\pi | x |}$$

C- Spread factor 0= σ

It is of the infinity activity type and it has infinity variation.

4- In order to compare investments in different securities, it is normal to look at the relative price variable over time and that S > 0 is defined as the dividends and it represents the variable relative price over time to compare investment $\frac{S_{t+s} - S_t}{S_t}$.

The logarithm of dividends within a single period of time is:

13)… $X_t = \log S_t - \log S_{t-1}$

X_t represents logarithm of the daily dividends.

The X_t dividends logarithm process is the same as in equation 2.11 where t = 1, which refers to the NIG- Lévy model.

$$X^{NIG} = \beta\delta^2 X^{IG} + \delta W_{X^{IG}} + M$$

As:

$X^{NIG} \sim$ NIG (α, β , δ, M)

As for the pricing asset model, it is as follows:

$$S_t = s_0 e^{X^{NIG}}$$

With the assumption that:

$$s_0 = 1$$

Then:

14)… $S_t = e^{X^{NIG}}$

2- Parameter estimation methods of Normal Inverse Gaussian (NIG) model:

Moments and Maximum Likelihood methods were used to find the estimations of model parameters:

2-3-1 Moments Method Estimation (MME)

By relying on the sample moments, moments estimation for NIG distribution density function were found in equation 5:

15)… $$\hat{\alpha} = \frac{\sqrt{3b_2 - 4\, b_1^2 - 9}}{m_2\left(b_2 - \frac{5}{3}\, b_1^2 - 3\right)^2}$$

16)…
$$\hat{\beta} = \frac{b_1}{\sqrt{m_2}\left(b_2 - \frac{5}{3} b_1^2 - 3\right)}$$

17)…
$$\hat{\delta} = \frac{3^{3/2} \sqrt{m_2\left(b_2 - \frac{5}{3} b_1^2 - 3\right)}}{3b_2 - 4 b_1^2 - 9}$$

18)…

$$\hat{M} = m_1 - \frac{3b_1 \sqrt{m_2}}{\left(3b_2 - 4 b_1^2 - 9\right)}$$

3-2 Maximum Likelihood Estimation

Fminsearch function was used in MATLAB program in order to find NIG model parameter estimations by using the maximum likelihood. This function uses Nelder –Mead method which selects the initial values in Fminsearch function by relying on MME.

III. THE PRACTICAL ASPECT

4-1 Data

A series of data were adopted for the closing price of the index in the Iraqi Stock Market for The North and The United Banks for the period 02/01/2014 until 16/11/2014.

4-2 NIG- Lévy model parameter estimation

Moments and maximum likelihood estimations were used to estimate model parameter as follows:

Table 3 shows NIG- Lévy model parameter estimation

M	δ	β	α	Method	
-0.003	0.0178	-6.683	17.4866	MME	North Bank
-0.0017	0.0103	-1.5577	7.3753	MLE	
-0.000143	0.0179	-3.6324	23.9094	MME	The United Bank
-0.0005	0.0121	-2.6351	15.6324	MLE	

The above table shows the estimated parameters of the Northern and United Banks through the parameter α it is note that the risk ratio represented by parameter α is higher for the United Bank compared to the North Bank, which means that the United Bank has a greater dividend than the North Bank because of the direct relation between dividend and risk.

As for parameter β, which shows the constant risk, we find that β is less with the United Bank compared to the Northern Bank.

As for the amount of fluctuations measured by parameter $\boldsymbol{\delta}$ it was lower with the Northern Bank compared to the United Bank and all the estimations used.

In order to provide useful information to investors and financial analysts that help them rationalize their investment decisions, the statistical characteristics of the NIG- Lévy model were found in Table 4 based on equations (15,16,17,18)

Table 4 shows the statistical characteristics of NIG- Lévy model

drift	**C.V**	**Kurt**	**Skew**	**Var**	**Mean**		**Number of circulation days**	
-0.0040	-3.4797	19.4236	-2.1223	0.0013	-0.004	MME	158	North Bank
-0.0039	-9.8511	50.6121	-2.3253	0.0015	-0.0039	MLE		
-0.0029	-9.6204	10.7468	-0.7008	$7.7717e^{-04}$	-0.0029	MME	185	The United Bank
-0.0026	-11.0652	20.9193	-1.1712	8.0824e-04	-0.0026	MLE		

It is clear from the statistical characteristics of the Northern and Unified Banks that the United Bank has a higher rate of dividend than the rate of dividend of the North Bank. The United Bank also received the least amount of variance and the least amount of kurtosis, due to the increase of the parameter α and the decrease of the parameter β. The United Bank has also obtained a greater skewness from the North Bank due to the impact of the parameter. The United Bank then obtained a lower c.v coefficient from the North Bank. Through drift parameter it can be observed that the United Bank has a shift during the trading year that is greater than the shift of the North Bank.

After that, the NIG- Lévy model is found based on equation 4. Figures 3 and 4 of the NIG- Lévy model for the North and the United Banks show the estimations of MME and MLE.

Figure 3 shows the comparison of dividends algorithm with the model at the MME and MLE of the North Bank

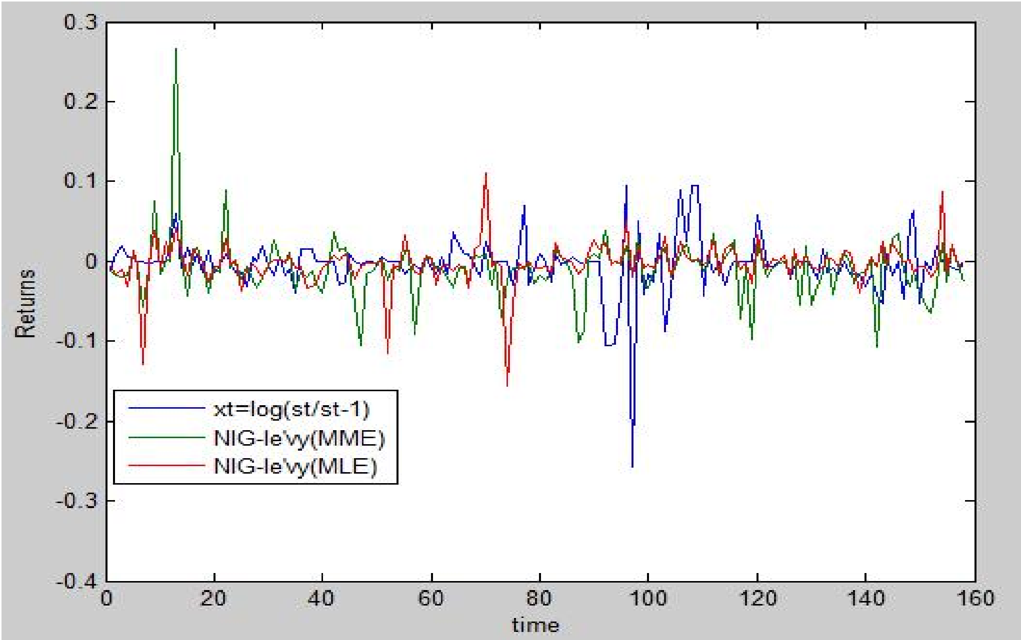
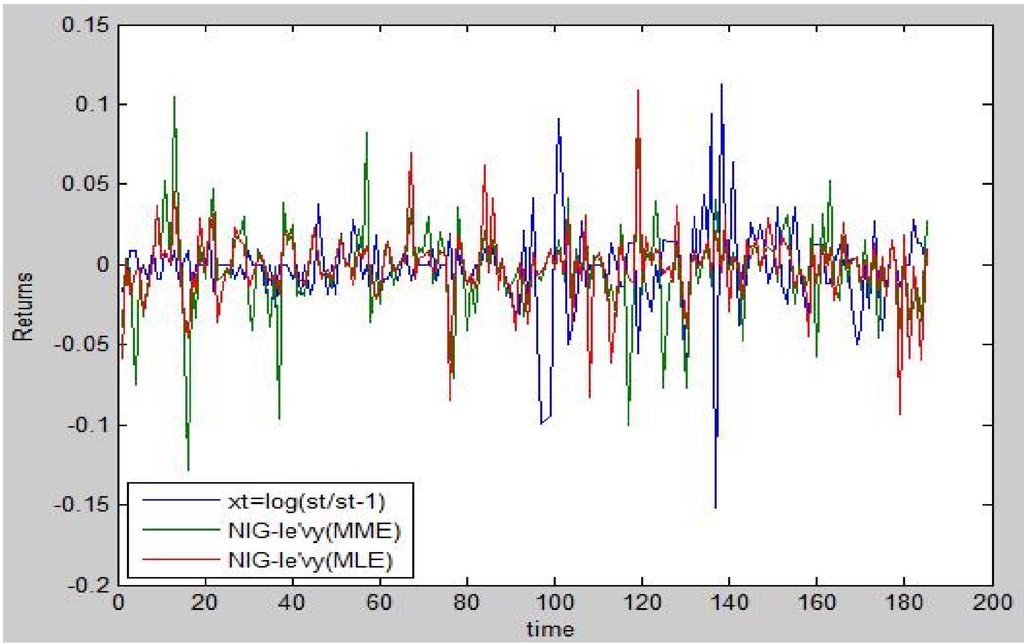


Figure 4 shows the comparison of the dividends algorithm with the model at the MME and MLE of the United Bank



From figures 4 and 3 we find that the MLE is close to dividends logarithm data than MME, this is clear by finding MSE for the model at MME and MLE in the following table.

Table 4 shows the average of error squares total of NIG- Lévy model

MSE		
0.0014	MME	North Bank

0.000717	MLE	
0.000844	MME	The United Bank
0.000548	MLE	

Table 4 shows that maximum likelihood estimation has less MSE compared to moments estimation. And the United Bank has less MSE than the MSE in the North Bank.

IV. THE EMPIRICAL ASPECT

5-1 Discussion of simulation trials

After obtaining the MLE and MME for NIG- Lévy in the practical aspect for the North and the United Banks, simulation trial results for MME and MLE of NIG- Lévy will be displayed and analysed in this subject.

Assuming that the sample sizes are (100,150,200) with a frequency of 500 times for each of the simulation trials that show the parameters of each bank separately by different sampling sizes and different parameters of that bank by taking hypothetical values for each parameter separately (except parameter M because it does not affect the pricing asset model) that may increase or decrease the estimations of the parameters obtained from the empirical aspect. The following is the interpretation of the kurtosis parameter as shown below:

Table 7 shows MASE values for NIG- Lévy with the assumption of the difference in sample sizes, the variance of parameter α and the constancy of the rest of the parameters of the North Bank when using MME and MLE

MLE	MME	α parameter value	Sample sizes
0.0651	0.0899	3.5	100
0.0035	0.0228	7.3753	
0.0026	0.0014	11	
0.0032	0.0065	12	
0.0053	0.0063	17.487	
2.8298e-4	0.0037	21.5	
0.0316	0.0206	3.5	150
0.0032	0.0042	7.3753	
0.0017	0.0108	11	
0.0038	0.0048	12	
0.0036	0.0092	17.487	
2.757e-4	0.0024	21.5	
0.0049	0.3791	3.5	200
0.0031	0.0045	7.3753	
7.8731e-4	0.0013	11	
0.0036	0.0185	12	
0.0030	0.0048	17.487	

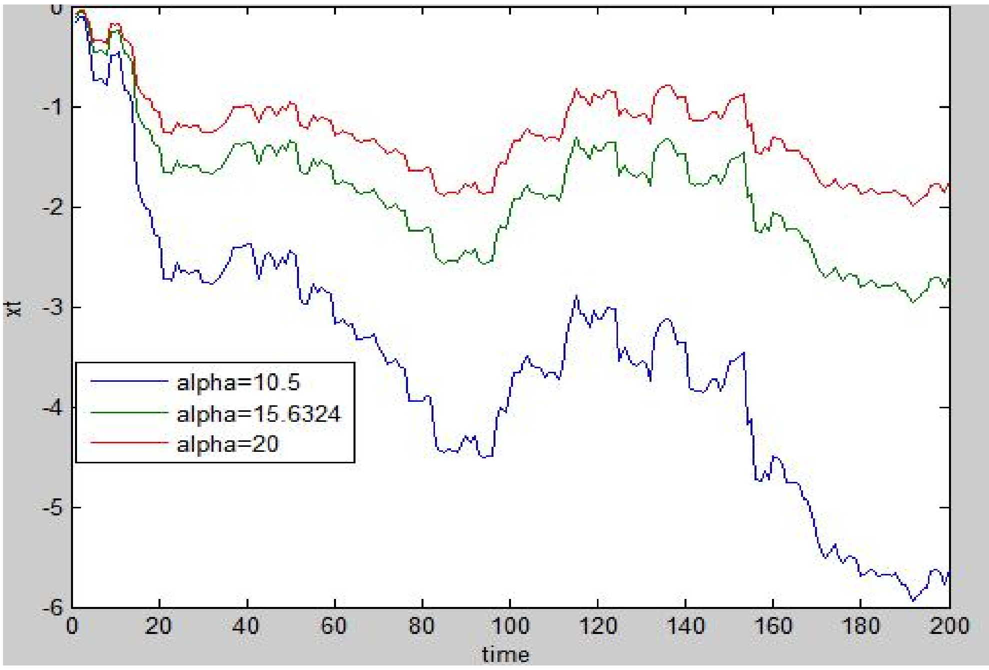
1.735e-4	0.0014	21.5	

Table 8 shows MASE values for NIG- Lévy with the assumption of the difference in sample sizes, the change of parameter α and the constancy of the rest of the parameters of the United Bank

MLE	MME	α Parameter value	Sample sizes
0.0136	0.0265	18.5	100
8.3836e-4	0.0019	15.632	
4.4746e-4	0.0012	20	
0.0084	0.0255	10.5	
9.6295e-4	0.0035	23.909	
3.7338e-4	5.5273e-4	28.5	
0.0095	0.0158	10.5	150
7.3211e-4	0.0022	15.632	
3.1242e-4	5.0735e-4	20	
0.0056	0.0087	18.5	
9.0323e-4	0.0013	23.909	
1.5321e-4	2.4747e-4	28.5	
0.0053	0.0113	10.5	200
6.7238e-4	2.5668e-4	15.632	
1.9073e-4	8.5932e-4	20	
0.0024	0.0115	18.5	
7.0614e-4	4.7441e-4	23.909	
1.3274e-4	1.4246e-4	28.5	

Figure 7 shows the simulation of NIG- Lévy model track for the United Bank at MLE with the variance of α value and the constancy of

$\beta =- 2.6351 \; \delta = 0.0121 \quad M =- 0.0005$



The figure above shows the significance of α parameter and the effect of NIG- Lévy model with the change of α value, it is noticed that there is a decrease in the large fluctuations in the model's track due to the increase of α parameter. This change in the value of α results in an increase in risk and consequently the increase in dividends due to the direct relationship between dividend and risk.

As can be seen from tables 7 and 8, MLE exceed MME because MLE have less MASE.

- MASE values decrease with the increase of sample value of MLE, while MASE value for MME were fluctuating.
- Decrease of MASE values with the increase of α kurtosis parameter at MLE, while for MME, MASE were fluctuating.
- At α single value MASE decrease with the increase of sample values especially MLE.
- Table 7 shows that the best value of parameter α was at (21.5 = α) for the Northern Bank. The value of MASE was the least compared to the other α values and for all sample sizes.
- As for Table 8, we note that it is better that (α = 28.5) value be for the United Bank because the increase in parameter α also led to the decrease in MASE values for all sample sizes.
- This is evident when the model track was drawn for one sample in figure 7 for the North and the United Banks where we notice that by the decrease of parameter α, the large fluctuations in the track decrease.

In order to explain the change in skewness parameter of NIG- Lévy model, MASE values were as follows:

Table 9 shows MASE values for NIG- Lévy model, with the assumption of the difference in sample sizes, change of parameter β and the constancy of the rest of the parameters of the North Bank.

MLE	MME	β parameter value	Sample sizes
0.034	0.0104	-2.5	100
0.0035	0.0228	-1.56	
0.0157	0.0103	-0.5	
8.7429e-4	0.109	-7.1	
0.0053	0.0063	-6.638	

0.0055	0.0044	-4.5	
0.003	0.0314	-2.5	150
0.0032	0.0042	-1.56	
0.0094	0.0039	-0.5	
5.894e-4	9.59112e-4	-7.1	
0.0036	0.0092	-6.638	
0.0048	0.0056	-4.5	
0.0011	0.0227	-2.5	200
0.0031	0.0045	-1.56	
0.0086	0.0109	-0.5	
2.2784e-4	0.005	-7.1	
0.003	0.0048	-6.638	
0.0034	0.0145	-4.5	

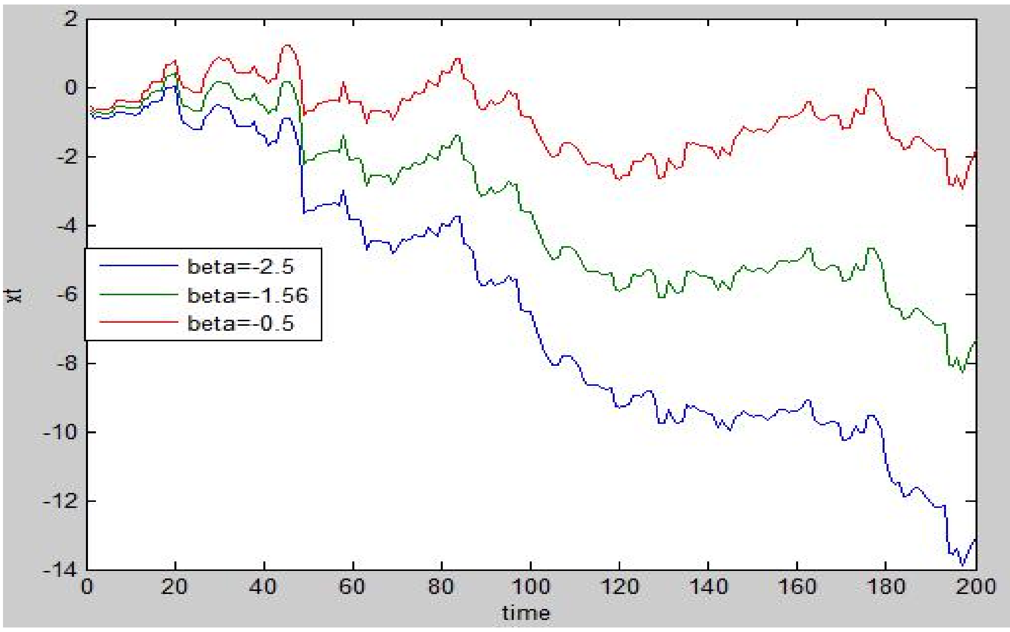
Table 10 shows MASE values for NIG- Lévy model, with the assumption of the difference in sample sizes, change of parameter β and the constancy of the rest of the parameters of the United Bank.

MLE	MME	β parameter value	Sample sizes
8.8649e-4	0.0015	-3.5	100
8.9836e-4	0.0019	-2.635	
0.0065	0.0025	0	
2.3616e-4	4.7799e-4	-4.1	
9.6295e-4	0.0035	-3.632	
9.7624e-4	9.9413e-4	2	
5.7189e-4	2.9144e-4	-3.5	150
7.3211e-4	0.0022	-2.635	
0.0035	0.0026	0	
1.3924e-4	2.574e-4	-4.1	
9.0323e-4	0.0013	-3.632	
9.4003e-4	8.7741e-4	2	
4.8103e-4	6.3809e-4	-3.5	200
6.7238e-4	2.5668e-4	-2.635	
0.0019	0.0039	0	
1.1912e-4	3.4416e-4	-4.1	
7.0614e-4	4.7441e-4	-3.632	
9.032e-4	3.1495e-4	2	

Figure 8 shows the simulation of NIG- Lévy model track for the North Bank with the assumption that

$\alpha = 7.3753 \;\; \delta = 0.0103 \;\;\; M = -\, 0.0017$

with the variance of β values



The figure above shows the significance of parameter β and the effect of NIG- Lévy model with the change of β value. It is noticed that there is a decrease in the large fluctuations in the track of the model due to the decrease in parameter β. The reason for the decline downwards is because of parameter β being negative.

From tables 9 and 10 it can be shown that MLE is superior to MME because MLE have less MASE.

- MASE values decrease with the increase of sample size of all MLE, while MASE values were fluctuating for MME.
- It can also be noticed that MASE values decrease when reducing skewness parameter values at MLE, while MASE values were fluctuating at MME.
- At β single value we find that MASE values decrease with the increase of sample sizes especially at MLE.
- In table 9 it is noticed that it is better that β =-7.1 for the North Bank.
- As for the United Bank, the results through table 10 have proven that the best value for parameter β was at $\beta = -\,4.1$. as MASE value was the least compared to the rest of β values and for all sample sizes.
- This is evident when the model track was drawn for one sample in figure 8 for the North and the United Banks where we notice that by the change of parameter β and the constancy of the rest of parameters when $\beta = 0$, the track will almost be similar. While when β is negative (positive), the track skewness will decline downwards (upwards). The decrease in β values lead to great fluctuations decrease.
- To explain the change of fluctuation parameter in NIG- Lévy model, MASE values were shown in the following tables:

Table 11 shows MASE values for NIG- Lévy model, with the assumption of the difference in sample sizes, change of parameter δ and the constancy of the rest of the parameters of the North Bank.

MLE	MME	β parameter value	Sample sizes
0.0046	0.0014	0.01	

0.0035	0.0228	0.0103	100
0.0046	0.0133	0.02	
5.4744e-4	0.0022	0.01	
0.0053	0.0063	0.0178	
3.293e-4	0.0035	0.03	
0.0031	0.0019	0.01	150
0.0032	0.0042	0.0103	
0.0037	0.0063	0.02	
3.569e-4	0.0013	0.01	
0.0036	0.0092	0.0178	
3.6578e-4	0.001	0.03	
0.0035	0.0191	0.01	200
0.0031	0.0045	0.0103	
0.0027	0.002	0.02	
7.7592e-5	0.0046	0.01	
0.0030	0.0048	0.0178	
3.9202e-4	0.0057	0.03	

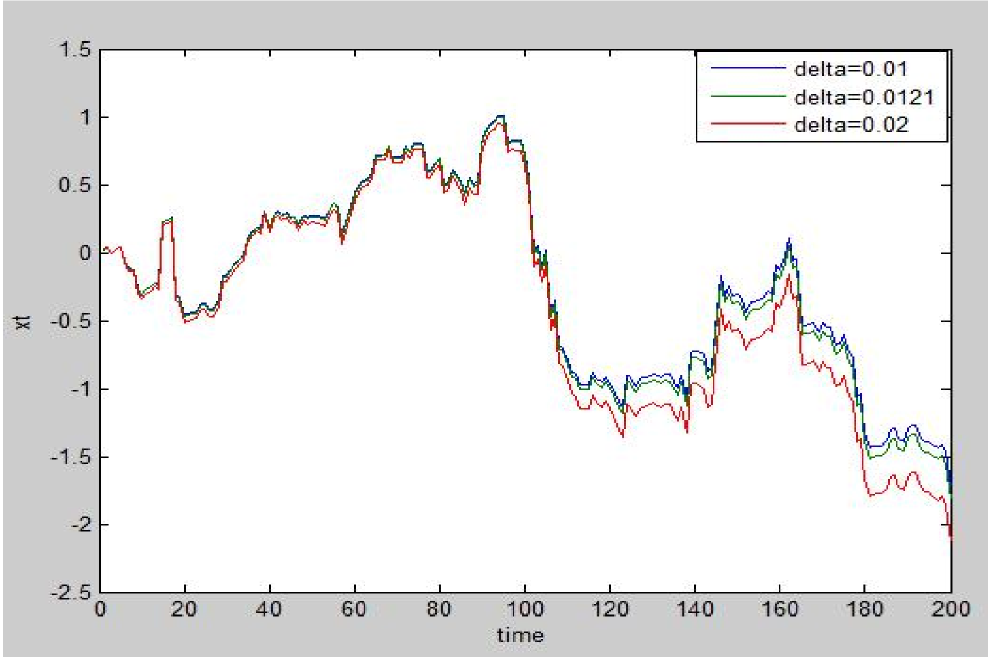
Table 12 shows MASE values for NIG- Lévy model, with the assumption of the difference in sample sizes, change of parameter δ and the constancy of the rest of the parameters of the United Bank.

MLE	MME	β parameter value	Sample sizes
8.9814e-4	0.0013	0.01	100
8.38836e-4	0.0019	0.0121	
3.306e-4	0.0034	0.02	
3.9692e-4	0.0012	0.01	
9.6295e-4	0.0035	0.0181	
6.5954e-4	9.7871e-5	0.02	
4.6525e-4	1.584e-4	0.01	150
7.3211e-4	0.0022	0.0121	
2.4723e-4	2.5366e-4	0.02	
2.029e-4	0.0013	0.01	
9.0323e-4	0.0013	0.0181	
4.6727e-4	5.4814e-4	0.02	
3.1665e-4	8.3223e-4	0.01	200
6.7238e-4	2.5668e-4	0.0121	
1.958e-4	0.0023	0.02	
1.5971e-4	8.0323e-5	0.01	
7.0614e-4	4.7441e-4	0.0181	

2.7891e-4	9.5607e-4	0.02	

Figure 9 shows the simulation of NIG- Lévy model track for the United Bank with the assumption that

$$\alpha = 15.6324 \quad \beta =- 2.6351 \quad \mathrm{M} =- 0.0005$$



From the figure above we noticed that the decrease of parameter δ has led to the increase of track activity. It can be shown from tables 11 and 12 MASE values when parameter δ changes with the constancy of the rest of the parameters

- MLE are better than MME for both Banks.
- In table 11 it can be noticed that the increases in parameter δ at sample size 100 is better than δ decrease. While decreasing δ has reduced MASE values at sample size 150,200 for the North Bank. Thus it can be said that decreasing δ is better than increasing it for the North Bank.
- As for the United Bank, table 12 shows that the increase in parameter δ at MLE has led to the reduction of MASE value at all sample sizes which means that the increase in parameter δ is better than decreasing it for the United Bank, on the contrary of the North Bank.
- This is clear when the track of one sample model was drawn in figure 9 for the North and the United Banks.
- Through the empirical aspect and after taking the estimated parameters obtained in the applied aspect and with different cases, it is found that MLE is superior to MME.

V. REFERENCES

[1]. Abramowitz, M. and I. A. Stegun (1968). Handbook of Mathematical Functions. New York: Dover Publ.

[2]. Anderson, T. W. (1958). An introduction to multivariate statistical analysis. New York: Wiley.

[3]. Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath (1997). Thinking coherently. Risk 10, 68–71. Artzner, P., F. Delbaen, J.-M. Eber, and D. Heath (1999). Coherent measures of risk. Mathematical Finance 9, 203–228.

[4]. Atkinson, A. C. (1982). The simulation of generalized inverse Gaussian and hyperbolic random variables. SIAM Journal on Scientific Computing 3, 502–515.

[5]. Bakshi, G., C. Cao, and Z. Chen (1997). Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

[6]. Bakshi, G., C. Cao, and Z. Chen (1998). Do call prices and the underlying stock move in opposite directions? Working paper, University of Maryland. To appear in Review of Financial Studies.