# Downtime Distribution for Analyzing Inflow and Outflow of Water in Mettur Dam 

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#### Abstract

Agriculture is completely based on irrigation and water as the output factor for food production .Large Dams plays a major role in lifting up the economic and social development of a country. The inflow and outflow of water from Dam plays a predominant role in the agriculture sector. In this paper we inspect the inflow and outflow of water in Mettur Dam during June 2008-May 2009, mathematically we fit the downtime distribution in terms of Laplace transforms (LT) and predicted the Laplace transforms is inverted numerically. Some properties of downtime distributions have also been reviewed.


Key words--Water Inflow and outflow, Laplace Transforms, Laplace steljtes transform, downtime, Poisson point process

## I. INTRODUCTION

Water is becoming a scare resource and the demand for water is increasing. Dams play a vital role in the conservation of water resources and also alters the distributions of water [8].The perfect usage of water from Dam requires a wide prediction of future availability of water resources. Let us take upon the effectiveness of inflow and outflow of water is dignified by the uptime and downtime (i.e) the total quantity of time the inflow and outflow of water is high or low during June 2008-May2009.Several authors had given cumulative distribution function (CDF) of total downtime in various methods, such as set theoretic method by Takacs[9]the method based on consideration of excess time by Muth[5],conditioning technique by Funaki and Yoshimoto[3].the expression for probability density function (PDF) is given by Srinivasan et al [9]the distribution of total downtime approaches a normal distribution [6][10], In all the above mentioned papers they had given independent variable ,in this paper we use a distinct method for deriving the distribution of total downtime ,Here we get into a dependence relation (i.e)the inflow and outflow of water are dependent .we derive total downtime as a functional of Poisson point process

## II. DISTRIBUTION OF TOTAL DOWNTIME

Let $\left(Y_{i}\right)$ and $\left(Z_{i}\right), i \geq 1$, signify the amount of inflow and outflow of water (i.e)it is either sufficient or not. The sequence $\left(Y_{i}, Z_{i}\right)$ of random vectors is i.i.d with sternly positive components. Here we set the random vectors to be more general than in [3],[5],[6],[9],[10] meanwhile we take $\left(Y_{i}\right)$ and $\left(Z_{i}\right)$ to be dependent, since the inflow and outflow of water are dependent .Let $S_{n}=\sum_{i=1}^{n}\left(Y_{i}+Z_{i}\right)$ for $n \geq 1$ and $S_{0} \equiv 0$, let

[^0]$N(t)=\sup \left\{n \geq 0: S_{n} \leq t\right\}$.Here when there is no adequate amount of inflow and outflow of water in Dam we represent it by downtime, and the total downtime is specified by,

$D(t)= \begin{cases}\sum_{i=1}^{N(t)} Z_{i} & \text { if } S_{N(t)} \leq t<S_{N(t)}+Y_{N(t)+1} \\ \left(\frac{1}{t}\right)^{-1}-\sum_{i=1}^{N(t)+1} Y_{i} & \text { if } S_{N(t)}+Y_{N(t)+1} \leq t<S_{N(t)+1}\end{cases}$
Represent the state of the system at time t by $X(t)$.if $X(t)$ is right continuous then the total downtime $\mathrm{D}(\mathrm{t})$ is specified by,

$$
\begin{equation*}
D(t)=\int_{0}^{t} 1_{\{0\}}(X(s)) d s \tag{2}
\end{equation*}
$$

The symbolization for CDFs is given as follows

$$
\begin{aligned}
& F(x)=P\left(y \geq Y_{1}\right) \\
& G(z)=P\left(z \geq Z_{1}\right) \\
& H(y, z)=P\left(y \geq Y_{1}, z \geq Z_{1}\right) \\
& K(w)=P\left(w \geq Y_{1}+Z_{1}\right)
\end{aligned}
$$

Here we designate $F_{n}$ and $G_{n}$ for the CDFs of $\sum_{i=1}^{n} Y_{i}$ and $\sum_{i=1}^{n} Z_{i}$ respectively. Let $F^{*}$ and $H^{*}$ signify the Laplace stieljies transforms of a CDF F and a joint CDF H.
For $\rho, \theta>0$

$$
\begin{aligned}
& F^{*}(\rho)=\int_{0}^{\infty} \frac{1}{e^{\rho y}} d F(y) \\
& H^{*}(\rho, \theta)=\int_{0}^{\infty} \int_{0}^{\infty} \frac{1}{e^{(\rho y+\theta z)}} d H(y, z)
\end{aligned}
$$

For the derivation of distribution of total downtime we use point process, Let ( $\Omega, \mathrm{F}, \mathrm{P}$ ) be the probability space on which i.i.d sequence $\left(Y_{i}, Z_{i}\right)$ is demarcated and also an i.i.d sequence $\left(U_{i}, i \geq 1\right)$ of exponentially distributed random variables with parameter 1 ,both the sequence are independent. Let $\left(T_{n}, n \geq 1\right)$ be the sequence of partial sums of the variables $U_{i}$, then the map
$\omega: \eta \rightarrow \sum_{n=1}^{\infty} \gamma_{\left(T_{n}(\eta), Y_{n}(\eta), Z_{n}(\eta)\right)}$

The dirac measure $\delta_{(y, z, x)}$ in $(\mathrm{y}, \mathrm{z}, \mathrm{x})$ defines a Poisson process on $E=[0, \infty) \times[0, \infty) \times[0, \infty)$ with intensity measure $v(d t d y d z)=d t d H(y, z)$ e.g.[7].For almost all $\eta \in \Omega, \omega(\eta)$ is a simple point measure on E such that there is atmost one point from the support (supp)of $\omega(\eta)$ on each set $\{t\} \times[0, \infty) \times[0, \infty)$. Let the set of all point measures on E is signified by $M_{p}(E)$.and the distribution of $\omega$ over $M_{p}(E)$ denoted by $P_{v}$.

For $t \in[0, \infty)$, define on $M_{p}(E)$ the functionals
$A_{Y}(t)(\mu)=\int_{E} y 1_{[0, t)}(s) \mu(d s d y d z)$
$A_{Z}(t)(\mu)=\int_{E} z 1_{[0, t)}(s) \mu(d s d y d z)$
$A(t)(\mu)=A_{Y}(t)(\mu)+A_{Z}(t)(\mu)$
For illustration $A(t(\mu))$ is the sum of the y and z coordinates of the points in the set of $\operatorname{supp} \mu \cap[0, t] \times[0, \infty) \times[0, \infty)$.where $\operatorname{supp} \mu=\{(s, y, z): \mu\{(s, y, z)\}>0\}$, we write $A_{Y}(t, \mu)$ for $A_{Y}(t)(\mu)$ and $A_{Z}(t, \mu)$ for $A_{Z}(t)(\mu)$, for $t \geq 0$

$$
B(t)(\mu)=\int_{t}\left\{1_{[0, y)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right) A_{z}(s, \mu)+1_{[y, y+z)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right)\left[t\left(1-\frac{A_{Y}(s+, \mu)}{t}\right)\right] \mu(d s d y d z)\right\}
$$

Where

$$
A_{Y}(s+, \mu)=\int_{E} y 1_{[0, s]}(r) \mu(d r d y d z)
$$

## Theorem1:

For $\rho, \theta>0$
$\int_{0}^{\infty} E\left[\frac{1}{e^{\rho D(t)}}\right] \frac{1}{e^{\theta t}} d t=\frac{\left.\rho \mid F^{*}(\theta)\left(\left(F^{*}(\theta)\right)^{-1}-1\right)\right]+\theta\left[H^{*}(\theta, \rho+\theta)\left(\left(H^{*}(\theta, \rho+\theta)\right)^{-1}-1\right)\right]}{\theta^{2}\left(\rho \theta^{-1}+1\right)\left[H^{*}(\theta, \rho+\theta)\left(\left(H^{*}(\theta, \rho+\theta)\right)^{-1}-1\right)\right]}$

## Proof:

By using With probability 1
$D(t)=B(t)(\omega)$
and Fubini's theorem, we get
$\int_{0}^{\infty} E\left[\frac{1}{e^{\rho D(t)}}\right] \frac{1}{e^{\theta t}} d t$

International Journal of Psychosocial Rehabilitation, Vol. 24, Issue 08, 2020
ISSN: 1475-7192

$$
\begin{aligned}
& =\int_{0}^{\infty} d t \int_{M_{p}(E)} P_{v}(d \mu) e^{\left\{-\rho \int_{E} \mu(d s d y d z)\right.}\left[1_{[0, y)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right) A_{z}(s, \mu)+1[y, y+z)\right. \\
& \left.\left.=\int_{M_{p}(E)} P_{v}(d \mu) \int_{E} \mu(d s d y d z) \int_{0}^{\infty} d t\left[1_{[0, y)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right)\left(t\left(1-\frac{A_{y}(s+, \mu)}{t}\right)\right)\right]\right]\right\} e_{e^{\frac{1}{t}}}^{t} \\
& =I_{1}+I_{2}
\end{aligned}
$$

Where
$I_{1}=\int_{M_{p}(E)} p_{v}(d \mu) \int_{E} \mu(d s d y d z) \int_{0}^{\infty} d t 1_{[0, x)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right) \frac{1}{e^{\rho A_{z}(s, \mu)}} \frac{1}{e^{\theta_{t}}}$
$I_{2}=\int_{M_{p}(E)} p_{v}(d \mu) \int_{E} \mu(d s d y d z) \int_{0}^{\infty} d t 1_{[y, y+z)}\left(t\left(1-\frac{A(s, \mu)}{t}\right)\right) \frac{1}{e^{\rho\left(t\left(1-\frac{A(s+, \mu)}{t}\right)\right)}} \frac{1}{e^{\theta t}}$
Using palm formula and Laplace functional of poison point process [4] and [7], we get

$$
\begin{align*}
& I_{1}=\theta^{-1} \int_{M_{p}(E)} p_{v}(d \mu) \int_{E} \mu(d s d y d z)\left[1-\frac{1}{e^{\theta_{y}}}\right] \frac{1}{e^{\rho A_{Z}(s, \mu)+\theta(s, \mu)}} \\
& =\theta^{-1} \int_{0}^{\infty} d s \int_{0}^{\infty} \int_{0}^{\infty} d H(y, z) \int_{M_{p}(E)} P_{v}\left(d \mu\left[1-\frac{1}{e^{\theta_{y}}}\right]\right) \frac{1}{e^{\rho A_{z}(s, \mu)+\theta(s, \mu)}} \\
& =\theta^{-1} \int_{0}^{\infty} \int_{0}^{\infty} d H(y, z)\left[1-\frac{1}{e^{\theta y}}\right] \int_{0}^{\infty} d s \int_{M_{p}(E)} P_{v}(d \mu) \times e^{\left\{-\int_{E} \mu(d \bar{s} d \bar{y} d \bar{z}) l_{[0, s)}(\bar{s})(\rho \bar{z}+\theta(\bar{y}+\bar{z}))\right\}} \\
& \left.=\theta^{-1}\left[F^{*}(\theta)\left(\left(F^{*}(\theta)\right)^{-1}-1\right)\right] \int_{0}^{\infty} d s e^{\left(-\int_{0}^{\infty} d \bar{s} \int_{0}^{\infty} \int_{0}^{\infty} d H(\bar{y}, \bar{z})\left[1-e^{-1[0, d)(\bar{s}(\sigma+(\rho+\theta) \bar{z}))}\right]\right.}\right] \\
& =\theta^{-1}\left[F^{*}(\theta)\left(\left(F^{*}(\theta)\right)^{-1}-1\right)\right] \int_{0}^{\infty} e^{\left(-s \int_{0}^{\infty} \int_{0}^{\infty} d H(\bar{x}, \bar{y})\left[1-\frac{1}{\left.e^{\left(\sigma \sigma^{(\rho+(\rho+\theta) \bar{z}}\right)}\right]}\right)\right.} \\
& I_{1}=\frac{\left|F^{*}(\theta)\left(\left(F^{*}(\theta)\right)^{-1}-1\right)\right|}{\theta\left[H^{*}(\theta, \rho+\theta)\left(\left(H^{*}(\theta, \rho+\theta)\right)^{-1}-1\right)\right]}  \tag{4}\\
& I_{2}=\int_{M_{p}(E)} P_{v}(d \mu) \int_{E} \mu(d s d y d z) \int_{y}^{y+z} d t e^{-(\rho+\theta) A(s, \mu)+\rho A_{Y}(s+, \mu)} \frac{1}{e^{(\rho+\theta) t}}
\end{align*}
$$

$$
\begin{align*}
& =(\rho+\theta)^{-1} \int_{0}^{\infty} d s \int_{0}^{\infty} \int_{0}^{\infty} d H(y, z) \int_{M_{p}(E)} P_{v}(d \mu)\left[\frac{1}{e^{(\rho+\theta) y}}-e^{-(\rho+\theta)(y+z)}\right] \times e^{\left\{-(\rho+\theta) A(s-\mu)+\rho\left[A_{y}(s+, \mu)+y\right]\right\}} \\
& =(\rho+\theta)^{-1} \int_{0}^{\infty} \int_{0}^{\infty} d H(y, z) \frac{1}{e^{\theta_{y}}}\left[1-\frac{1}{e^{(\rho+\theta) z}}\right]_{0}^{\infty} d s \int_{M_{p}(E)} P_{v}(d \mu) \times e\left\{-\int_{E} \mu(d \bar{s} d \bar{y} d \bar{z})\left[1_{[0, s)}(\bar{s})(\rho+\theta)(\bar{y}+\bar{z})-1_{[0, s)}(\bar{s}) \rho \bar{y}\right]\right\} \\
& \left.=(\rho+\theta)^{-1}\left[F^{*}(\theta)-H^{*}(\theta, \mu+\theta)\right] \int_{0}^{\infty} d s \times e^{\left\{-\int_{0}^{\infty} d s \int_{00}^{\infty} \int_{0}^{\infty} d H(\bar{y}, \bar{z}) \times\left[1-e^{(-[[[0, s)(\bar{s})(\rho+\theta)(\overline{\bar{y}}+\bar{i})-[(0, s)(\overline{( }) \bar{j})]}\right]\right.}\right] \\
& =(\rho+\theta)^{-1}\left[F^{*}(\theta)-H^{*}(\theta, \rho+\theta)\right] \int_{0}^{\infty} d s \times e^{\left\{-s \int_{0}^{\infty}\left[\int_{0}^{\left[1-\frac{1}{\left.e^{(\sigma+t}+\rho+\theta\right) \bar{y}}\right]}\right] d H(\bar{y}, \overline{\bar{z}})\right\}} \\
& \left.I_{2}=\frac{F^{*}(\theta)-H^{*}(\theta, \rho+\theta)}{(\rho+\theta)\left[H^{*}(\theta, \rho+\theta)\left(\left(H^{*}(\theta, \rho+\theta)\right)^{-1}-1\right)\right.}\right] \tag{5}
\end{align*}
$$

## Corollary1:

Taking derivatives with respect to $\rho$ in (3) and $\rho=0$ we get

$$
\begin{align*}
& \int_{0}^{\infty} E[D(t)] \frac{1}{e^{\theta_{t}}}=\frac{\theta^{-2}\left[F^{*}(\theta)-H^{*}(\theta, \theta)\right]}{1-H^{*}(\theta, \theta)}  \tag{6}\\
& \int_{0}^{\infty} E\left[D(t)^{2}\right] \frac{1}{e^{\theta t}} d t=2 \theta^{-3}\left[\frac{F^{*}(\theta)-H^{*}(\theta, \theta)}{1-H^{*}(\theta, \theta)}-\frac{\theta\left[1-F^{*}(\theta)\right] E\left(Y_{1} \frac{1}{e^{\theta\left(Y_{1}+Z_{1}\right)}}\right)}{\left[1-H^{*}(\theta, \theta)\right]^{2}}\right] \tag{7}
\end{align*}
$$

## III. ASSYMPTOTIC PROPERTIES OF D (t)

In this we use a method of takacs [10] which is based on a assessment with the asymptotic properties of a delayed renewal process associated to the process that we remark in this paper.

$$
\begin{array}{ll}
\mu_{Y}=E\left(Y_{1}\right), & \mu_{Z}=E\left(Z_{1}\right) \\
\sigma_{Y}^{2}=\operatorname{var}\left(Y_{1}\right) & \sigma_{Z}^{2}=\operatorname{var}\left(Z_{1}\right) \quad \sigma_{y z}=\operatorname{cov}\left(Y_{1}, Z_{1}\right)
\end{array}
$$

## Theorem2:

If $\mu_{Y}+\mu_{Z}<\infty$,then

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \frac{E[D(t)]}{t}=\frac{\mu_{Z}}{\mu_{Y}+\mu_{Z}} \tag{8}
\end{equation*}
$$

If $\sigma_{Y}^{2}$ and $\sigma_{Z}^{2}$ are finite and $Y_{1}+Z_{1}$ is a non-lattice random variable, then

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ISSN: 1475-7192
$\lim _{t \rightarrow \infty}\left(E[D(t)]-\frac{\mu_{Z} t}{\mu_{Y}+\mu_{Z}}\right)=\frac{\mu_{Z} \sigma_{Y}^{2}-\mu_{Y} \sigma_{Z}^{2}-2 \mu_{Y} \sigma_{Y Z}}{2\left(\mu_{Y}+\mu_{Z}\right)^{2}}-\frac{\mu_{Y} \mu_{Z}}{2\left(\mu_{Y}+\mu_{Z}\right)}$
and
$\lim _{t \rightarrow \infty} \frac{\operatorname{var}[D(t)]}{t}=\frac{\mu_{Y}^{2} \sigma_{Z}^{2}+\mu_{Z}^{2} \sigma_{Y}^{2}-2 \mu_{Y} \mu_{Z} \sigma_{Y Z}}{\left(\mu_{Y}+\mu_{Z}\right)^{3}}$
Proof:
Let $\bar{N}(t)$ be the delayed renewal process determined by the random variables $\left(V_{n}\right), \mathrm{n}=0,1,2, \ldots$. , where $V_{0}$ has the distribution $P\left(V_{0} \leq y\right)=\left(1 / \mu_{Y}\right) \int_{0}^{y}[1-F(z)] d z$ for $y \geq 0$ and $P\left(V_{0} \leq y\right)=0$
$V_{n}=Y_{n}+Z_{n} \quad n=1,2,3, \ldots \ldots \ldots$.
Using (6) and Laplace stieltjes arguments we can prove that

$$
\begin{equation*}
E[D(t)]+\mu_{Y} E[\bar{N}(t)]=t \tag{11}
\end{equation*}
$$

Since $\lim _{t \rightarrow \infty} E[\bar{N}(t)] / t=1 / \mu_{Y}+\mu_{Z}$
To ascertain (9) from (11)we obtain
$\lim _{t \rightarrow \infty}\left(E[D(t)]-\frac{\mu_{Z} t}{\mu_{Y}+\mu_{Z}}\right)=-\mu_{Y} \lim _{t \rightarrow \infty}\left(E[\bar{N}(t)]-\frac{t}{\mu_{Y}+\mu_{Z}}\right)$
If $Y_{1}+Z_{1}$ is a non-lattice random variable, using formula (33)of [10] and $E\left(V_{0}\right)=\left(\sigma_{Y}^{2}+\mu_{Y}^{2}\right) / 2 \mu_{Y}$, we get the proof of (9).

To ascertain asymptotic variance of $\mathrm{D}(\mathrm{t})$ we assume $\bar{N}(t)$ be the delayed renewal process determined by the random variables $\left(V_{n}\right), \mathrm{n}=0,1,2, \ldots$..where $V_{0}$ has LT
$E\left(\frac{1}{e^{\theta V_{0}}}\right)=\frac{\left.\left[F^{*}(\theta)\left(\left(F^{*}(\theta)\right)^{-1}-1\right)\right)\right]_{0}^{\infty} \int_{0}^{\infty} z \frac{1}{e^{\theta(y+z)}} d H(y, z)}{\theta \mu_{Y} \mu_{Z}}$
and $V_{n}$ for $n \geq 1$ is defined as in (10).then using corollary 1 and formula (30)and(31) of [10] we can prove that

$$
\begin{equation*}
E\left[D(t)^{2}\right]=2 \int_{0}^{t} E[D(u)] d u-\mu_{y} \mu_{z}\left(E[\bar{N}(t)]+E\left[\bar{N}(t)^{2}\right]\right) \tag{12}
\end{equation*}
$$

If $Y_{1}+Z_{1}$ is a non-lattice random variable, using formula (33),(39)of [10] and using above equation as $t \rightarrow \infty$, we get

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ISSN: 1475-7192

$$
E\left[D(t)^{2}\right]=\frac{\mu_{Z}^{2} t^{2}}{\left(\mu_{Y}+\mu_{Z}\right)^{2}}-\left[\frac{\mu_{Y} \mu_{Z}^{3}+\left(\mu_{Y}^{2}-2 \sigma_{Y}^{2}\right) \mu_{Z}^{2}+\left(\sigma_{Z}^{2}+4 \sigma_{Y Z}\right) \mu_{Y} \mu_{Z}-\mu_{Y}^{2} \sigma_{Z}^{2}}{\left(\mu_{Y}+\mu_{Z}\right)^{3}}\right] t+o(t)
$$

By using (9) the last part of theorem is followed

## IV. ASSYMPTOTIC DISTRIBUTION OF d(T)

## Theorem3:

If $\sigma_{Y}^{2}$ and $\sigma_{Z}^{2}$ are finite then

$$
\frac{D(t)-\mu_{Z} t /\left(\mu_{Y}+\mu_{Z}\right)}{\sqrt{\left(\left(\mu_{Y}^{2} \sigma_{Z}^{2}+\mu_{Z}^{2} \sigma_{Y}^{2}-2 \mu_{Y} \mu_{Z} \sigma_{Y Z}\right) /\left(\mu_{Y}+\mu_{Z}\right)^{3}\right)}} \stackrel{D}{\longrightarrow} N(0,1) \text { as } t \rightarrow \infty
$$

Proof:
$\sum_{i=1}^{N(t)} Z_{i} \leq D(t) \leq \sum_{i=1}^{N(t)+1} Y_{i}$
Where $N(t)=\operatorname{su}\left\{n \geq 0: \sum_{j=1}^{n}\left(Y_{j}+Z_{j}\right) \leq t\right\}$
By means of central limit theorem for random sums [2] we get

$$
\frac{1}{\sqrt{\frac{\mu_{Y}^{2} \sigma_{Z}^{2}+\mu_{Z}^{2} \sigma_{Y}^{2}-2 \mu_{Y} \mu_{Z} \sigma_{Y Z}}{\left(\mu_{Y}+\mu_{Z}\right)^{3}}}}\left(\sum_{i=1}^{N(t)} Z_{i}-\frac{\mu_{Z}(t)}{\mu_{Y}+\mu_{Z}}\right) \xrightarrow{D} N(0,1)
$$

The proof is complete if we can show that
$\left(Z_{N(t)+1}\right) t^{-\frac{1}{2}} \xrightarrow{D} 0 \quad$ as $t \rightarrow \infty$
$N(t) / t \xrightarrow{p} 1 /\left(\mu_{Y}+\mu_{Z}\right)$
By means of this fact and assuming $\sigma_{Z}^{2}<\infty$ we obtain $Z_{N(t)} / \sqrt{N(t)} \xrightarrow{p} 0$ [6].
Here for an illustration we consider both $Y_{1}$ and $Z_{1}$ are exponentially distributed with parameters $\lambda+v$ and $\mu+v$

Hence $\mu_{Y}=\frac{1}{\lambda+v} \mu_{Z}=\frac{1}{\mu+v}$
$\sigma_{Y}^{2}=\frac{1}{(\lambda+v)^{2}} \sigma_{Z}^{2}=\frac{1}{(\mu+v)^{2}}$
The covariance and correlation coefficient between $Y_{1}$ and $Z_{1}$ are given by

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ISSN: 1475-7192

$$
\sigma_{Y Z}=\frac{v}{(\lambda+\mu+v)(\lambda+v)(\mu+v)} \quad \text { and } \quad \beta_{Y Z}=\frac{v}{\lambda+\mu+v}
$$

The total downtime $\mathrm{D}(\mathrm{t})$ using 3 is given by

$$
\int_{0}^{\infty} E[D(t)] \frac{1}{e^{\theta t}} d t=\frac{\theta^{-2}[(2 \lambda+v) \theta+(\lambda+v)(\lambda+\mu+v)]}{2 \theta^{2}+(3 \lambda+3 \mu+4 v) \theta+(\lambda+\mu)(\lambda+\mu+3 v)+2 v^{2}}
$$

## V. SIMULATION STUDY

We inspect the inflow and outflow of water in Mettur Dam from June 2008- May2009.Firstly we sight out the inflow and outflow of water in each month, then we calculate the mean value for inflow and outflow of water for every 15 days in each month. So while computing these mean values we get 24 sets of data and is signified in Table 1.

Table1: Average Inflow And Outflow Of Water In Mettur Dam

|  | Month | Inflow | Outflow |
| :---: | :---: | :---: | :---: |
| 1 | June 1-15 | 3898.40 | 3255.27 |
| 2 | June 16-30 | 1060.00 | 12704.37 |
| 3 | Jul 1-15 | 1253.93 | 13956.83 |
| 4 | Ju116-31 | 2405.63 | 12579.81 |
| 5 | Aug 1-15 | 28230.00 | 12857.19 |
| 6 | Aug 16-31 | 23464.50 | 13880.41 |
| 7 | Sep 1-15 | 22102.00 | 13974.60 |
| 8 | Sep 16-30 | 8535.03 | 18940.97 |
| 9 | Oct 1-15 | 10150.97 | 18222.77 |
| 10 | Oct 16-31 | 18533.88 | 3877.72 |
| 11 | Nov 1-15 | 6983.73 | 14715.57 |
| 12 | Nov 16-30 | 8436.07 | 5134.40 |
| 13 | Dec 1-15 | 9283.03 | 1430.67 |
| 14 | Dec 16-31 | 3483.72 | 5499.75 |
| 15 | Jan1-15 | 957.70 | 10001.17 |
| 16 | Jan 16-31 | 369.70 | 5843.21 |
| 17 | Feb 1-14 | 485.11 | 1622.18 |
| 18 | Feb 15-28 | 646.21 | 1600.50 |
| 19 | Mar 1-15 | 649.03 | 1470.50 |
| 20 | Mar 16-31 | 814.59 | 1123.34 |

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ISSN: 1475-7192

| 21 | Apr 1-15 | 1392.53 | 1360.67 |
| :--- | :--- | :--- | :--- |
| 22 | Apr 16-30 | 1010.43 | 1349.60 |
| 23 | May 1-15 | 529.90 | 1744.57 |
| 24 | May16-31 | 3171.69 | 854.97 |

For the data specified in the above table we plot out the graph for inflow and outflow of water and is signified in Fig. 1


Fig.1, Graph representing inflow and outflow of water in Mettur Dam using departmental data
In Fig. 1 the horizontal axis represents the interval of 15 days, the vertical axis represents the flow of water in cusecs. We apply the points obtained from the graph in SPSS software and obtained the various distributions values and applying it to the downtime Laplace equation we get a different 24 sets of values for the inflow and outflow of water related to the distributions, then we again plot the graph for these values we get Fig. 2 (i.e.)mathematical graph.


Fig.2, Graph plotted mathematically using downtime distribution

## VI. CONCLUSION

Water is becoming one of the abundant resource in the upcoming century, the agriculture also lost its importance due to uncultivated lands, and the land loses its fertility due to scarcity of water. In our inspection of inflow and outflow of water in Mettur Dam during June 2008-May 2009 ,the total inflow is nearly 204 TMC.The water is to be released from Mettur Dam during the month of June for irrigation ,but the inflow of water in the Dam is insufficient and level of water in the Dam is also not sufficient .Only by the month of August 2008 the inflow of water is given in adequate amount and level reaches a minimum level for releasing water from Dam and water is released for irrigation .


Fig. 3 Predicted graph for inflow and outflow of water
Sometimes it is seen that inflow of water was surplus and is drop out to sea .From the inflow of 204 TMC they can provide inflow of water at an average of nearly 12 TMC for every 15 days from the month of June 2008 to January2009 as given in Fig.3,that is approximately 1 TMC(11000 Cusec ) per day from June 12,2008 to January 31,2009 for nearly 234 days ,so that water would be released from Mettur Dam during the month of June .so cultivation would be started during the month of June which in turn increases the agriculture production .the adequate inflow of water increases groundwater in and around the areas of Mettur Dam and kabini,Krishna raja sagara Dam .Here in this paper we fit out the downtime distribution for inadequate inflow and outflow of water in Dam, derived out the distributions and properties of downtime distribution fitted it with the inflow and outflow of water .Finally, the graph plotted using departmental data exactly fits with the graph plotted using mathematical data.

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