# ALGORITHMS FOR IDENTIFICATION OF LINEAR DYNAMIC CONTROL OBJECTS BASED ON THE PSEUDO-CONCEPT CONCEPT 


#### Abstract

${ }^{1}$ Zaripov O.O., ${ }^{2}$ Shukurova O.P., ${ }^{3}$ Sevinov J.U. Abstract: Regularized algorithms for identifying linear dynamic control objects based on the concept of pseudo inversion are given. The equation of the control object is focused on solving a number of applied problems and is selected in the form of a multidimensional dynamic regression equation. To solve the equation in question, we use differentiation formulas for quadratic functionals with respect to matrix variables and regularized algorithms based on non-orthogonal factorizations and pseudo in versions of square matrices. The obtained regular algorithms contribute to increasing the accuracy of estimating the parameters of the class of dynamic control objects under consideration.


Keywords: linear dynamic control objects, identification algorithms, regular estimation, matrix pseudo inverse

## I. INTRODUCTION

Currently, in connection with the development of technology, the complexity of managed objects in developed and designed control systems is significantly increased. The structure of most modern control objects is such that the exact mathematical description of the objects is either absent or varies widely. In such conditions, the incompleteness of information about the mathematical model imposes a significant limitation on the methods used for the synthesis of models and controls. One of the decisive steps in the identification process is the choice of the class of mathematical objects in which a representative is selected that is most suitable for some criterion of a real simulated dynamic system [1-11]. Since one of the main directions of the practical use of models is predicting the output signals of a real technological system, it is natural, when assessing the quality of conformity of a model to a real object, to take the result of comparing their outputs as a criterion for conformity.

## II. FORMULATION OF THE PROBLEM <br> Consider a linear discrete dynamical system described by the equation

[^0]\[

$$
\begin{equation*}
y(t+1)=\sum_{i=0}^{p} \mathrm{~A}_{i} y(t-i)+\sum_{j=0}^{q} \mathrm{~B}_{j} \mathrm{u}(t-j)+\xi(t), \quad t=1,2, \ldots, N, \tag{1}
\end{equation*}
$$

\]

where $\mathrm{y}(\mathrm{t}), \mathrm{u}(\mathrm{t})$ are state and control vectors of dimension $(l, 1),(m, 1) ; A_{i}, i=0,1, \ldots, p, B_{j}, j=0,1, \ldots, q$ .The functioning of the system is carried out for $t=0,1,2, \ldots, N$ based on management $\mathrm{u}(0), \mathrm{u}(1), \ldots, \mathrm{u}(N-1)$; initial states are set $y(0), y(-1), \ldots, y(-\mathrm{p})$ and initial management $\mathrm{u}(-1), \ldots, \mathrm{u}(-q)$.

To solve the identification problem, difference vectors are written $\Delta(t, y, v, A, B)$

$$
\Delta(t, y, v, A, B)=y(t+1)-\sum_{i=0}^{p} A_{i} y(t-i)-\sum_{j=0}^{q} B_{j} v(t-j)-\xi(t)
$$

and the functional is formed $q(y, v, A, B)[2,3]$

$$
\begin{equation*}
q(y, v, A, B)=\sum_{t=0}^{N-1} \Delta^{T}(t, y, v, A, B) \Delta(t, y, v, A, B) \tag{2}
\end{equation*}
$$

where $y, v, A, B$ - designations of sequences of vectors and matrices: $y=(y(-p), y(-p+1), \ldots, y(N))$, $v=(v(-q), v(-q+1), \ldots, v(N-1)), A=\left(A_{1}, \ldots, A_{p}\right), B=\left(B_{1}, \ldots, B_{q}\right)$. Matrices: $A^{0}=\left(A_{0}^{0}, A_{1}^{0}, \ldots, A_{p}^{0}\right)$, $B^{0}=\left(B_{0}^{0}, B_{1}^{0}, \ldots, B_{q}^{0}\right)$, minimizing (2) are taken as a solution to the identification problem

$$
\left(A^{0}, B^{0}\right)=\arg \left\{\min _{A, B} q(y, v, A, B)\right\}
$$

It can be seen that (2) is a quadratic form of the elements of the matrices $A_{i}, B_{j}$ and is a system of linear equations.

We use the following notation:

$$
\begin{gathered}
\varphi_{\bar{i}}(t), \bar{i}=1, \ldots, N_{M}, N_{M}=p+1+q+1+1 \\
\varphi_{1}(t)=y(t), \varphi_{2}(t)=y(t-1), \ldots, \varphi_{p+1}(t)=y(t-p) \\
\varphi_{p+2}(t)=v(t), \ldots, \varphi_{p+q+2}(t)=v(t-q), \varphi_{N_{M}}(t)=1
\end{gathered}
$$

Introducing the notation $\varphi_{1}(t)=y(t), \varphi_{2}=v(t), \varphi_{3}=1$ дляр $=q=0, N_{M}=3$ you can write the following expressions:

$$
\begin{equation*}
G_{\bar{i}, \bar{j}}=\sum_{t=0}^{N-1} \varphi_{\bar{i}}(t) \varphi_{\bar{j}}^{T}(t), D_{\bar{i}}=\sum_{t=0}^{N-1} y(t+1) \varphi_{\bar{i}}^{T}(t), \bar{i}, \bar{j}=1,2,3 \tag{3}
\end{equation*}
$$

Given (3) and based on the structure of functional (2), we can write the following expressions for the matrices $G_{\bar{i}, \bar{j}}$ и $D_{\bar{i}}, \bar{i}, \bar{j}=1, \ldots, N_{M}$ :

$$
\begin{equation*}
G_{\bar{i}, \bar{j}}=\sum_{t=0}^{N-1} \varphi_{\bar{i}}(t) \varphi_{\bar{j}}^{T}(t), D_{\bar{i}}=\sum_{t=0}^{N-1} y(t+1) \varphi_{\bar{i}}^{T}(t) \tag{4}
\end{equation*}
$$

Then one can form block matrices $\boldsymbol{G}, \boldsymbol{D}$, and $\boldsymbol{X}$ as follows [3]: the matrix $\boldsymbol{G}$ is composed of $N_{M}^{2}$ matrices $G_{\bar{i}, \bar{j}}^{T}$; matricesD is made up of $N_{M}$ matrices $D_{\bar{i}}^{T}$, located by column; matrix $X$ composed of $N_{M}$ matrices $A_{0}^{T}, \ldots, A_{p}^{T}, B_{0}^{T}, \ldots, B_{q}^{T}$, disposable column. Matrices $\boldsymbol{G}, \boldsymbol{D}$, Xhave dimensions $\left(N_{q p}, N_{q p}\right),\left(N_{q p}, l\right),\left(N_{q p}, l\right)$, where $N_{q p}=l(p+1)+m(q+1)+1$.

$$
G=\left(\begin{array}{c:c:c:c}
G_{11}^{T} & G_{12}^{T} & \cdots & G_{1, N_{M}}^{T}  \tag{5}\\
\hdashline G_{21}^{T} & & & \\
\hdashline G_{31}^{T} & & & \\
\hdashline \vdots & \vdots & \vdots & \vdots \\
\hdashline \vdots & \vdots & \vdots & \vdots \\
\hdashline \vdots & \vdots & \vdots & \vdots \\
\hdashline G_{N_{M}, 1}^{T} & \cdots & \cdots & G_{N_{M}, N_{M}}^{T}
\end{array}\right), \quad D=\left(\begin{array}{c}
D_{1}^{T} \\
\hdashline D_{2}^{T} \\
\hdashline D_{3}^{T} \\
\hdashline \vdots \\
\hdashline- \\
\hdashline \vdots \\
\hdashline- \\
\hdashline \vdots \\
D_{N_{M}}^{T}
\end{array}\right), \quad X=\left(\begin{array}{c}
A_{0}^{T} \\
-\vdots \\
\hdashline \\
\hdashline A_{p}^{T} \\
-B_{0}^{T} \\
\hdashline \vdots \\
\hdashline \\
B_{q}^{T} \\
-C
\end{array}\right) .
$$

Thus, the identification algorithm in this case is reduced to finding matrices $G_{\bar{i}, \tilde{j}}, D_{\bar{i}}$ on formulas(4); the formation of block matrices $G, D(5)$; calculating the matrix parameters of the system forming the block matrix X , based on the solution of the matrix system

$$
G X=D
$$

decaying into l systems of dimension $N_{q p}$ :

$$
\begin{equation*}
G x_{k}=d_{k}, \quad k=1,2, \ldots, l \tag{6}
\end{equation*}
$$

## III. DECISION BASED ON THE REGULAR METHOD

The system of equations (6) can be poorly conditioned, i.e. small changes to the source data may respond to large changes to the solution. The aforementioned circumstance when solving equation (6) leads to the necessity of using regularization methods [9, 12, 13].

In addition, in practical problems, elements, for example, matrices, are often known to us approximately. In these cases, instead of a matrix, we are dealing with some other matrix $\tilde{G}$ such that $\|\tilde{G}-G\| \leq h$, where the meaning of the norms is usually determined by the nature of the task. Having instead of matrix $G$ matrix $\tilde{G}$, all the more so, we cannot make a definite judgment on the degeneracy or non-degeneracy of the matrix $G$. But such matrices $\tilde{G}$ infinitely many, and within the framework of the level of error known to us, they are indistinguishable. Among such "possible exact systems" there may be degenerate ones.

To give numerical stability to the matrix inversion procedure, it is advisable to use the concepts of regular and stable estimation methods [12, 13]. Below is an algorithm for estimating the inverse matrix $G^{-1}$ in equation (6).

Let the matrix $G=\left[g_{i j}\right], i, j=1,2, \ldots, n$, non-degenerate. Denote by $G_{k}$ its upper left part, i.e.. $G_{k}=\left[g_{i j}\right], i, j=1,2, \ldots, k$. Matrices $G_{k}, k=1,2, \ldots, n$, non-degenerate. We represent them in the form

$$
G^{(k)}=\left[\begin{array}{c:c}
G_{11}^{(k-1)} & G_{12}^{(k)} \\
\hdashline G_{21}^{(k)} & G_{22}^{(k)}
\end{array}\right], k>1, G_{11}^{(1)}=G_{22}^{(1)}=g_{11}, \quad G_{12}^{(1)}=0, \quad G_{21}^{(1)}=0
$$

Following $[14,15]$ of the bordering method, the inverse matrix can be written in the following form

$$
\left[G^{(k)}\right]^{-1}=\left[\begin{array}{c:c}
P^{(k-1)} & R^{(k)} \\
\hdashline N^{(k)} & \overline{V^{(k)}}
\end{array}\right]
$$

while

$$
\begin{gather*}
V^{(k)}=\left[G_{22}^{(k)}-G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)}\right]^{-1} \\
R^{(k)}=-V^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)}, N^{(k)}=-V^{(k)} G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1}  \tag{7}\\
P^{(k-1)}=\left[G_{11}^{(k-1)}\right]^{-1}+V^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)} G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1}
\end{gather*}
$$

Performing calculations by formulas (7) at, we can obtain $G_{n}^{-1}=G^{-1}$.
In the case under consideration, it is advisable to use the Gauss method [14,16-18] to invert the matrix, according to which a sequence of matrices is constructed $G^{(k)}, k=0,1, \ldots, G^{(0)}=G$, on

$$
G^{(k)}=\left[\begin{array}{c:c}
G_{11}^{(k)} & G_{12}^{(k)} \\
\hdashline G_{21}^{(k)} & G_{22}^{(k)}
\end{array}\right],
$$

где $G_{11}^{(k)}$ - обратимая матрица размера $k \times k$.
Для этого на $k$-м шаге у имеющейся клетки $G_{22}^{(k)}$ ищется ведущий $\tau S$-й элемент

$$
\left|g_{\tau 5}^{(k)}\right|=\max _{k<i \leq n, k<j \leq m}\left|g_{i j}^{(k)}\right|,
$$

where $g_{i j}^{(k)}$ ifij-йэelement of matrix $G^{(k)}$
If $\left|g_{\tau \mathrm{s}}^{(k)}\right|>\varepsilon$, then permutation is done $\tau$-йstrings ands- th column with $(k+1)-$ mi row and column matrix $G^{(k)}$. Then it is recalculated according to the cell obtained after rearrangements

$$
G_{22}^{(k)}=\left[\begin{array}{c:c}
g_{k+1, k+1}^{(k)} & d^{(k)} \\
\hdashline \vartheta^{(k)} & W^{(\bar{k})}
\end{array}\right]
$$

matrix $G^{(k+1)}$, while

$$
G^{(k+1)}=\left[\begin{array}{c:c}
G_{11}^{(k)} & G_{12}^{(k)} \\
G_{21}^{(k)} & g_{k+1, k+1}^{(k)} \\
\hdashline \alpha^{(k)} & d_{22}^{(k)}
\end{array}\right],
$$

где $G_{22}^{(k+1)}=W^{(k)}-\alpha^{(k)} d^{(k)}, \alpha^{(k)}=\left(g_{k+1, k+1}^{(k)}\right)^{-1} \vartheta^{(k)}$.
If $\left|g_{\tau s}^{(k)}\right| \leq \varepsilon$, then the factorization process is terminated and non-orthogonal matrix factorization is determinedGin the form

$$
\begin{gathered}
G_{\varepsilon}=U_{k} R_{k}, U_{k}^{T}=\left[U_{1}^{T}: U_{2}^{T}\right], \\
R_{k}=\left[R_{1} \vdots R_{2}\right], U_{2}=G_{21}^{(k)}, \\
R_{2}=G_{12}^{(k)}, \quad U_{1}=\left[\begin{array}{cccc}
1 & 0 & \ldots & 0 \\
g_{21}^{(k)} & 1 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
g_{k 1}^{(k)} & g_{k 2}^{(k)} & \ldots & 1
\end{array}\right], R_{1}=\left[\begin{array}{cccc}
g_{11}^{(k)} & g_{12}^{(k)} & \ldots & g_{1 k}^{(k)} \\
0 & g_{22}^{(k)} & \ldots & g_{2 k}^{(k)} \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & g_{k k}^{(k)}
\end{array}\right] .
\end{gathered}
$$

The approximation for a pseudoinverse matrix is constructed in this way [13,17]:

$$
\begin{equation*}
G_{\varepsilon}^{+}=R_{k}^{+} U_{k}^{+} \tag{8}
\end{equation*}
$$

Calculation $R^{+}$and $U^{+}$in (8) when $R$ and $U^{T}$ accordingly, upper trapezoidal matrices are effectively carried out by orthogonal factorization $R=S P$ using Givens or Householder transformations [15,17], whereS - lower triangular square, $P$ - orthogonal matrix. Then $R^{+}=P^{T} S^{-1}$.

## IV. Conclusion

The presented regularized identification algorithms based on non-orthogonal factorizations and pseudoinversions of square matrices increase the accuracy of estimating the parameters of the class of dynamic control objects under consideration and predicting the output variables of a real technological system.

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